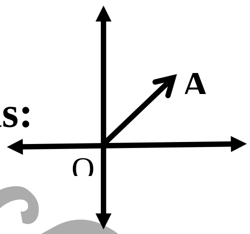
Rectilinear motion

Position vector:

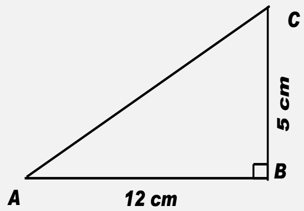


For any orthogonal coordinate system the position vector OA is:
The vector which its initial point is origin O and its end is A

Displacement vector: Is the shortest distance between two points

Distance: it is the real distance between two points

Example 1 In the opp. Fig:
A body moves from A to B then change its direction to C
Find the distance and the displacement during its motion



Solution: Distance = AB + BC = 5 + 12 = 17 m

Displacement = $\overline{AC} = \sqrt{25+144} = 13m$

Directrion : $\tan\theta = \frac{5}{12} \rightarrow \therefore \theta = 22^\circ 37^/ 11^{//}$

Relation between position vector and displacement vector:

If $r_1(x_1, y_1)$ *is a position vector at time* t_1

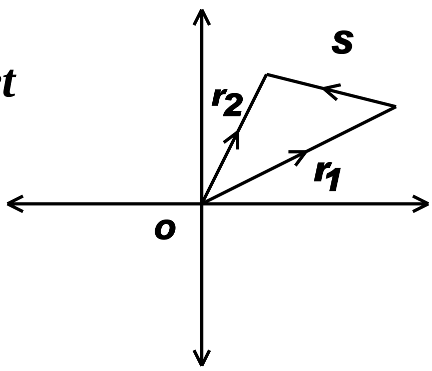
If $r_2(x_2, y_2)$ *is a position vector at time* t_2

From the triangular rule to sum vectors we get

$\overline{r_1} + \overline{s} = \overline{r_2} \quad \Rightarrow \therefore$ *Displacement* $\vec{s} = \vec{r}_2 - \vec{r}_1$

$\therefore s = r_2 - r_1 = (x_2, y_2) - (x_1, y_1)$

$\therefore \therefore \left\|\vec{s}\right\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Example 2: A particle moves so that its position vector $\vec{r}$ is given as a function in time in terms of the fundamental unit vectors $\vec{i}, \vec{j}$ with the relation:
$r = (t+3)i + (3t-2)j$
Find (1) displacement vector (2) displacement till time = 4 sec.

Solution:

$$\therefore \vec{s} = \vec{r}_2 - \vec{r}_1 = \left[(t+3)\vec{i} + (3t-2)\vec{j}\right] - \left[(3)\vec{i} - (2)\vec{j}\right]$$

$$S = (t+3)\vec{i} + (3t-2)\vec{j} - 3\vec{i} + 2\vec{j}$$

$$\therefore \text{Displaecment vector is } \bar{s} = t\ \bar{i} + 3t\ \bar{j}\ (\text{first})$$

$$\text{at } t = 4 \quad \Rightarrow \quad \vec{s} = 4\ \vec{i} + 12\ \vec{j}$$

$$\therefore \left\|\vec{s}\right\| = \sqrt{(4)^2 + (12)^2} =$$

Velocity vector

It is the vector which its magnitude equals the value of velocity and its direction is the same as motion direction

Units of measuring velocity km/hr , m/sec , cm/sec

Converting

$$(x)km/hr = (x) \times \frac{5}{18} m/\sec$$

$$(x)km/hr = (x) \times \frac{250}{9} cm/\sec$$

Average velocity

$$\text{Average velocity} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{vector of average velocity} = \frac{\text{displacement}}{\text{total time}}$$

Example 3

A cyclist covered 30 km on a straight road with velocity 18 km/hr., and then he returned on the same road and covered 20 km in the opposite direction with velocity 15 km/hr. Find the average velocity and the average velocity vector during the whole journey.

Solution

Example 4

If a particle takes two positions A(5,2) and B(9, 10) between two successive moments 3sec. and 7sec. respectively. Find the direction of the average velocity of the particle during this time interval, then find the magnitude and the direction of this average velocity.

Solution

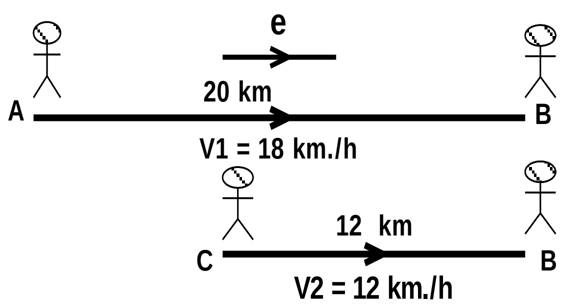
Example 5

A cyclist cut 20 km. on a st. road with a velocity 18 km./h. he returned and cut 12 km. in the opposite direction at 12 km./h. Find his average velocity direction during the whole trip.

Solution

$Time\ taken\ from\ A\ to\ B = \frac{S_1}{V_1}$

$= \frac{20}{18} = 1.11\ hours$



$Time\ taken\ from\ B\ to\ C = \frac{S_2}{V_2}$

$= \frac{12}{12} = 1\ hour$

$Total\ Time = 1.11 + 1 = 2.11\ hours$

$Total\ displacement\ \vec{S} = 20\ \hat{e} - 12\ \hat{e} = 8\ \hat{e}$

$\therefore \vec{V}_a = \frac{Total\ displacement}{Total\ Time} = \frac{8\ \hat{e}}{2.11} = 3.8\ \hat{e}$

$\therefore \vec{V}_a$ *vector has the same direction of* $\hat{e}$ *and its norm is about* $3.8\ km./h$

It is important to know that the norm of the total displacement is 8 km. but the whole distance is 32 km.

Example 6 *A train covered the distance between Cairo and Alex. in two stages*

First stage : From cairo to Tanta. the covered distance was 100 km. and the velocity was 100 km./h.

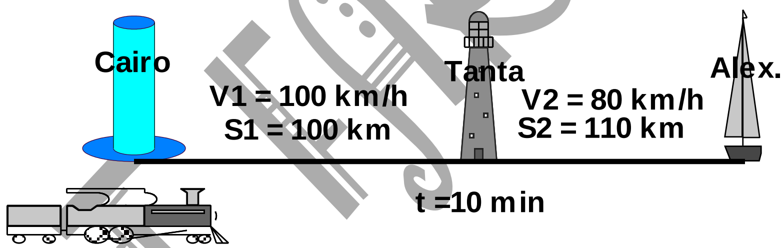
Second stage : from Tanta to Alex. the covered distance was 110 km. and the velocity was 80 km./h.

the stopping of the train in Tanta was 10 minits.

Find the average velovity of the train during its whole trip.

conceder that the train was moving the whole distance in St.line

Solution:



$$T_1 \ (Cairo \ to \ Tanta) = \frac{S_1}{V} = \frac{100}{100} = 1h$$

$$T_{1=2} \ (Tanta \ to \ Alex) = \frac{S_2}{V_2} = \frac{110}{80} = \frac{11}{8}h$$

$$10 \min = \frac{1}{6h}$$

$$Total \ \ Time \ = 1 + \frac{11}{8} + \frac{1}{6} \ = 2.54 \ hours$$

$$Total \ \ displacement \ \vec{S} = 100 \ \hat{e} - 110 \ \hat{e} = 210 \ \hat{e}$$

$$\therefore \vec{V}_a = \frac{Total \ \ displacement}{Total \ \ Time} = \frac{210 \ \hat{e}}{2.54} = 82.6 \ \hat{e}$$

The relative Velocity

$\vec{V}_{ab} = \vec{V}_a - \vec{V}_b$ $\Rightarrow$ *(velocity of a w.r.t b) " b watching a"*

$\vec{V}_{ba} = \vec{V}_b - \vec{V}_a$ $\Rightarrow$ *(velocity of b w.r.t a) " a watching b"*

Important remarks to solve problems

1) You must determine unite vector ($\vec{n}$) has positive direction with (a or b)

2) The different cases of motion of two bodies (a and b)

1) in the same direction : $V_{ab} = (V_a - V_b)\vec{n}$

2) in opposite direction : $V_{ab} = (V_a + V_b)\vec{n}$

3) If $Va \ = \ V_b$ **Then** $V_{ab} = 2V_a = 2V_b$

Important remarks to solve problems

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Ballistic motion حركة المقذوفات (Rocket, torpedo and shoot ....)

(1) The velocity of extruded (C) " p ser htiw " سرعة الشئ المقذوف ect to (B)

$V_{cb} = (V_a + V_c - B_b)\vec{n}$

(2) If A and B in opposite direction : thn velocity of C (w.r.t) B is

$V_{cb} = (V_a + V_c + B_b)\vec{n}$

(3) $S = V_{cb} \times t$

(4) Don't forget the directions of A, B, C (w.r.t) direction unit vector $\vec{n}$

Traines motion حركة القطارات

(1) in the same direction : $V_{ab} = (V_a - V_b)\vec{n}$

(2) To find time to " cross the bridge, cross one of them the other, ...)

$$t = \frac{S_1 + S_2}{|V_a - V_b|}$$

where S_1 , S_2 are the lengths of two trains

(1) in opposite direction : $V_{ab} = (V_a + V_b)\vec{n}$

(2) To find time to " cross the bridge, cross one of them the other, ...)

$$t = \frac{S_1 + S_2}{V_a + V_b}$$

where S_1 , S_2 are the lengths of two trains

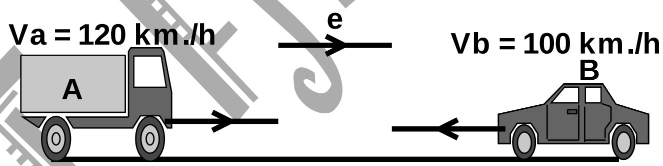
Example 7 A car "A" moves on a sraight with velocity 120 km./h.meets another car "B" moving with velocity 100 km./h. Find the velocity of "B" relative to "A" in case of
(1) B moves in an opposite direction of A
(2) B moves in the same direction as A

Solution

Let $\hat{e}$ is the unit vector, "in the same direction of A"
(1) B moves in an opposite direction of A

$\vec{V}_a = 120\ \hat{e}$, $\vec{V}_b = -100\ \hat{e}$

$\therefore \vec{V}_{ba} = \vec{V}_b - \vec{V}_a = -100\ \hat{e} - 120\ \hat{e} = -220\ \hat{e}$

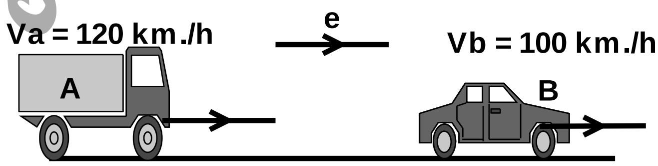


so that the car (A) will feel that car (B) is moving towards him at velocity 220 km./h

(2) B moves in the same direction as A

$\vec{V}_a = 120\ \hat{e}$, $\vec{V}_b = 100\ \hat{e}$

$\therefore \vec{V}_{ba} = \vec{V}_b - \vec{V}_a = 100\ \hat{e} - 120\ \hat{e} = -20\ \hat{e}$



so that the car (A) will feel that car (B) is moving towards him at velocity 20 km./h

Example 8 A controlling speed car (Radar) moves on the desert road (Cairo - Alex.) at 30 km./h. it watched a truck coming from the other opp. road which second moving at 110 km./h. Find the real velosity of the truck

Solution

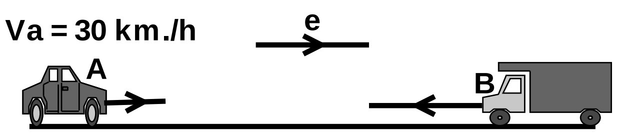
$\vec{V}_a = 30 km./h$, $\vec{V}_{ba} = 110 km./h$

$\therefore \vec{V}_{ba} = \vec{V}_b + \vec{V}_a$ (in the case of opp. directions)

$\vec{V}_b = \vec{V}_{ab} - \vec{V}_a$

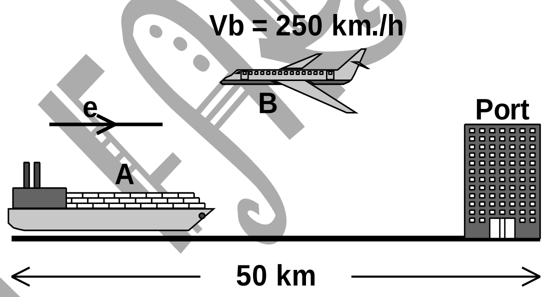
$\vec{V}_b = 110\ \hat{e} - 30\ \hat{e} = 80\ \hat{e}$

$V_b = 80 km./h$



Example 9 *A steamer was moving in st. way towards a port. at 50 km.from it, and aerolane passed over it in the opposite direction and at 250 km. It observed the steamer which seemed to the aeroplane as it is moving with 275 km./h, Find the time deposite from the moment the aeroplane observed the steamer till reach the port.*

Solution



$\vec{V}_b = 250\ \hat{e}$, $\vec{V}_{ab} = 275\ \hat{e}$

$\therefore \vec{V}_{ab} = \vec{V}_a + \vec{V}_b$ *(in the case of opp. directions)*

$\vec{V}_a = \vec{V}_{ab} - \vec{V}_b$

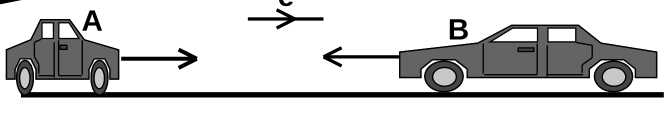
$\vec{V}_a = 275\ \hat{e} - 250\ \hat{e} = 25\ \hat{e}$

$V_b = 25 km./h$

$S = 50km. \Rightarrow T = \frac{S}{V} = \frac{50}{25} = 2 hours$

Example 10 *A car (A) moving on st. way measured the relative velocity of another car (B) coming from the opposite direction, it was 140 km./h when the car (A) reduced its velocity to its half and remeasured the relative velocity of car (B) it found it 120 km./h., Find the actual velocity of the two cars.*

Solution



$\vec{V}_{ba} = 140\ \hat{e}$ $\Rightarrow$ *(in case 1)*

$\vec{V}_{ba} = 120\ \hat{e}$ $\Rightarrow$ *(in case 2)*

(in case 1) (in the case of opp. directions)

$\therefore \vec{V}_{ba} = \vec{V}_b + \vec{V}_a \quad \Rightarrow 140 = \vec{V}_b + \vec{V}_a \rightarrow (1)$

(in case 2) (in the case of opp. directions)

$\therefore \vec{V}_{ba} = \vec{V}_b + \vec{V}_a$

$\Rightarrow 120 = \vec{V}_b + \frac{1}{2}\vec{V}_a \xrightarrow{\times 2} 240 = 2\vec{V}_b + \vec{V}_a \rightarrow (2)$

From (1) & (2) by subtracting we get

$V_b = 100 km./h$, $V_a = 40 km./h$

__Example__ 11 Two aeroplanes fly at the same speed in a st. line while one of them is following the other with distance 500 m., at certain moment the one dehind launched a rocket at the leading one. this rocket reached the first one after 2 sec. of its launching. what is the velocity of the rocket.?

Solution

the two aeroplanes having the same speed and fly at st. line then they are seemingly stoped, So we only will find the speed of a rocket $V=\frac{S}{t}=\frac{500}{2}=250\ m/\sec.$

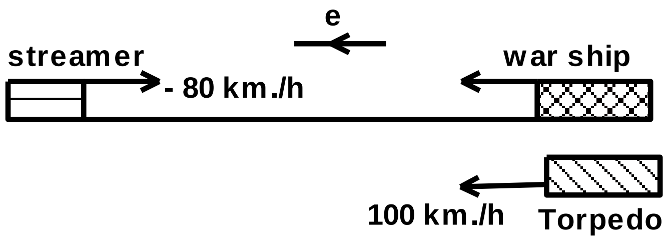
__Example__ 12 A war ship and a steamer are moving in a st. line at the same velocity, every one of them was moving towards the other. the war ship watching the motion of the steamer it seemed to it that it is moving at 80 km./h. when the distance between them became 6 km. the war ship launched a torbedo on the steamer if the engin of the torpedo can puch it 100 km./h. What is the time taken from the moment of the lounching and the moment the torpedo reaches the steamer

To find the time we use $t=\frac{S}{V}=\frac{6}{????}$, *So we need to find the velocity, But which velovity?.... velocity of the torpedo w.r.t. steamer*

So, $\therefore \boxed{\vec{V}_{ca}=\vec{V}_c-\vec{V}_a \xrightarrow{\text{by add } \vec{V}_b \& -\vec{V}_b} (\vec{V}_c-\vec{V}_b)-(\vec{V}_a-\vec{V}_b)} \rightarrow (1)$

Now we need to find $(\vec{V}_c-\vec{V}_b)$ & $(\vec{V}_a-\vec{V}_b)$

$\boxed{\vec{V}_{ab}=(\vec{V}_a-\vec{V}_b)=-80\ \hat{e}} \rightarrow (2)$



$\because \vec{V}_{cb}=100\ \hat{e}$ *(from given)*

$\boxed{\vec{V}_{cb}=(\vec{V}_c-\vec{V}_b)=100\ \hat{e}} \rightarrow (3)$

From (1),(2) and (3)

$\therefore \vec{V}_{ca}=(\vec{V}_c-\vec{V}_b)-(\vec{V}_a-\vec{V}_b)=(100-(-80))\ \hat{e}=180\ \hat{e}$

then $\Rightarrow t=\frac{S}{V}=\frac{6}{180}=\frac{1}{3}hr=20\min$

Example 13 A train moves with velocity 84 km./h. catches another one of length 120 m. and moves with velocity 60km./h. parallel to the first one. it completely leaves the second after 45 sec. Find the length of the first train, then find the time it takes to across a bridge of length 520 m. knowing that the second train moves in the same direction as the first

Solution

$\vec{V}_a = 84\ \hat{e}$, $\vec{V}_b = 60\ \hat{e}$

$\vec{V}_{ab} = \vec{V}_a - \vec{V}_b = 84\ \hat{e} - 60\ \hat{e} = 24\ \hat{e}$

the first train cuts $(S+120)m.$ with velocity $24\ km./h.$ in 45 sec

$So,\ 24\ km./h. = 24 \times \frac{5}{18} = \frac{20}{3}$ sec

$\because S = V \times T \quad \Rightarrow \therefore (S+120) = 24 \times \frac{20}{3}$

$S + 120 = 300 \quad \Rightarrow \therefore S = 180\ m.$

To the first train passes the bridge, it must cover a distance $180 + 520 = 700\ m.$

with velocity $84\ km./h = 48 \times \frac{5}{18} = \frac{70}{3}\ m/\sec$

$\therefore T = \frac{S}{V} = \frac{7000}{\frac{70}{3}} = 30$ sec

Example 4: A train of length 150 metres, moves with velocity 72 km./h. another train of length 100 meters is moving parallel to the first one. the first one catch the second. Find the time, that the first train completely passes the second, if the second train.

(1) stoped

(2) Moves with velocity 45 km./h in the same direction of first train

(3) Moves with velocity 45 km./h in opposite direction of first train

Solution

(1) $\overrightarrow{V}_a = 72\ \hat{e}$, $\overrightarrow{V}_b = \vec{0}$

$\overrightarrow{V}_{ab} = \overrightarrow{V}_a - \overrightarrow{V}_b = 72\ \hat{e} - \vec{0} = 72\ \hat{e}$

To the first train completely passes the second, it must cover a distance 150 + 100 = 250 m.

with velocity 72 km./h $= 72 \times \frac{5}{18} = 20\ m/\sec$

$\therefore T = \frac{S}{V} = \frac{250}{20} = 12.5\ \sec$

(2) $\overrightarrow{V}_a = 72\ \hat{e}$, $\overrightarrow{V}_b = 45\ \hat{e}$

$\overrightarrow{V}_{ab} = \overrightarrow{V}_a - \overrightarrow{V}_b = 72\ \hat{e} - 72\ \hat{e} = 27\ \hat{e}$

To the first train completely passes the second, it must cover a distance 150 + 100 = 250 m.

with velocity 72 km./h $= 27 \times \frac{5}{18} = 7.5\ m/\sec$

$\therefore T = \frac{S}{V} = \frac{250}{7.5} = 33\frac{1}{3}\ \sec$

(3) $\overrightarrow{V}_a = 72\ \hat{e}$, $\overrightarrow{V}_b = -45\ \hat{e}$

$\overrightarrow{V}_{ab} = \overrightarrow{V}_a - \overrightarrow{V}_b = 72\ \hat{e} - (-72\ \hat{e}) = 117\ \hat{e}$

To the first train completely passes the second, it must cover a distance 150 + 100 = 250 m.

with velocity 72 km./h $= 117 \times \frac{5}{18} = \frac{65}{2}\ m/\sec$

$\therefore T = \frac{S}{V} = \frac{250}{\frac{65}{2}} = \frac{100}{13}\ \sec$

Example 15: A train moves with velocity 84 km./h. catches another one of length 120 m. and moves with velocity 60 km./h. parallel to the first one. it completely leaves the second after 45 sec. Find the length of the first train, then find the time it takes to across a bridge of length 520 m. knowing that the second train moves in the same direction as the first

Solution

$\overrightarrow{V}_a = 84\ \hat{e}$, $\overrightarrow{V}_b = 60\ \hat{e}$

$\overrightarrow{V}_{ab} = \overrightarrow{V}_a - \overrightarrow{V}_b = 84\ \hat{e} - 60\ \hat{e} = 24\ \hat{e}$

the first train cuts $(S+120)m.$ with velocity $24\ km./h.$ in 45 sec

$So,\ 24\ km./h. = 24\times\frac{5}{18} = \frac{20}{3}$ sec

$\because S = V\times T \quad \Rightarrow \therefore (S+120) = 24\times\frac{20}{3}$

$S+120 = 300 \quad \Rightarrow \therefore S = 180\ m.$

To the first train passes the bridge, it must cover a distance $180 + 520 = 700\ m.$

with velocity $84\ km./h = 48\times\frac{5}{18} = \frac{70}{3}\ m/\text{sec}$

$\therefore T = \frac{S}{V} = \frac{7000}{\frac{70}{3}} = 30$ sec

Homework

Complete the following:

1. 20m/sec = ... km/hr
2. 90 km/hr = ... m/sec
3. A car moves with a uniform velocity of magnitude 72 km/hr for a quarter of an hour, then the covered distance = ... km.
4. If $\overrightarrow{V_A} = 15\ \overrightarrow{i}$, $\overrightarrow{V_B} = 22\ \overrightarrow{i}$ $\therefore\ \overrightarrow{V_{BA}} =$ ....
5. If $\overrightarrow{V_{AB}} = 65\ \overrightarrow{n}$, $\overrightarrow{V_A} = 50\ \overrightarrow{n}$ $\therefore\ \overrightarrow{V_B} =$ ....
6. A cyclist (A) moves on a straight road with a velocity of 15 km/hr. If he met another cyclist (B) moves with a velocity of 12 km/hr, then the velocity of B with respect to A equals .... km/hr.

Choose the correct answer:

7. If a car moves with uniform velocity 75 km/hr for 20 minutes, then the covered distance equals .... km

a) 15 b) 20 c) 25 d) 30

8. A car covered a distance of 180 km. With velocity 20 m/sec on a straight road, then the time taken to cover this distance = ... hours

   a) $1\frac{1}{2}$ b) - 2 c) $2\frac{1}{2}$ d) 3

9. If $\overrightarrow{V_{AB}} = 15\ \overrightarrow{i}$, $\overrightarrow{V_A} = 35\ \overrightarrow{i}$ $\therefore\ \overrightarrow{V_B}$ equals:

   a) $-50\ \overrightarrow{i}$ b) $-20\ \overrightarrow{i}$ c) $20\ \overrightarrow{i}$ d) $50\ \overrightarrow{i}$

10. If the position vector of a particle moves in a straight line from a point and gives a function in time t by the relation: $\overrightarrow{r} = (2t^2 + 3)\ \overrightarrow{n}$ then the magnitude of position vector $\overrightarrow{S}$ its norm is measured by meter after 2 seconds equal:

    a) 4m b) 6m c) 8m d) 11m

11. **Join with space:** If the sun light reaches the earth in 8.3 min. and the distance between the earth and the sun equals 1.494×10^{11} meter , find the velocity of the light.

12. Two cars moves at the same time from Banha towards Cairo with a constant velocity for each of them. If the velocity of the first car equals 70 km\hr and the velocity of the second car equals 84 km\hr .Find the taken time by the driver of the second car to reach the first car at the end of the trip whose length 49 km?



13. A train of length 150 meter entered a straight tunnel of length S meter. It took the entire crossing of the tunnel in a time of 15 seconds. Find the length of the tunnel if the train moves with uniform velocity equals 90 km/hr.

14. A cyclist covered 30 km on a straight road with a velocity of 15 km/hr, then he returned back and covered 10 km in the opposite direction with a velocity of 10 km/hr. Find the average velocity during his whole trip.

15. A traveller moved on a straight road, he covered 800 meters with velocity 9 km/hr, and then he returned back and covered the same distance in the same direction with velocity 4.5 km/hr., Find the magnitude of the average velocity of the traveller during the whole trip.

16. The distance between two cities A and B is 120 km. A car moved from the city A towards the city B with a velocity of 88 km/hr. At the same moment , another car moved from the city B towards the city A with a uniform velocity of 72 km/hr. Find when and where do the two cars meet.

17. A car (A) moves on a straight road with a uniform velocity 60 km/hr., If another car (B) moves with a uniform velocity of 90 km/hr. on the same road. Find the velocity of car (A) relative to the car (B) if:

    a) The two cars are moving in opposite direction.

    b) The two cars are moving in the same directions.

18. A police car moves in a straight line with a uniform velocity, if it recorded the relative velocity of a truck moves in its direction in front of it which equals 60 km/hr. ,If the police car doubles its velocity, and recorded the relative velocity of a truck again which seems to be at rest. Find the real velocity of each of the police car and the truck.

Rectilinear motion with uniform acceleration

Acceleration is the rate of change with respect to the time and denoted by (a)

$$\vec{a} = \frac{d\vec{v}}{dt} \quad , \quad \vec{a} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1}$$

Types of motion

1) **Uniform motion:** The velocity is constant (a = 0)
2) **Variable motion:** velocity changes with time acceleration is exist and not constant
3) **Uniform variable motion:** velocity changes regularly (acceleration is constant)

Equations of the uniform Variable motion in straight line

1] $V = V_\circ + at$ — Relation between velocity and time

2] $S = V t_\circ + \frac{1}{2} a t^2$ — Relation between distance and time

3] $V^2 = V_\circ^2 + 2aS$ — Relation between velocity and distance

Important remarks to solve problems

1) **Acceleration is positive when the velocity increasing and negative if velocity is decreasing**
2) **If the driver (car) uses the break then acceleration is negative**
3) **If the velocity is uniform then a = 0**
4) **If the velocity is constant then a = 0**
5) **If the body move with maximum velocity then a = 0**
6) **If the body start its motion from rest then V = 0**
7) **If the acceleration is in the same direction of motion then (a) is positive**
8) **If the acceleration is in opposite direction of motion then (a) is negative**
9) **Average velocity of a partical during time interval = its velocity at the middel of this time interval for example if a body covered a distance 6 m during fifth second Then:** $\frac{6}{1} = V_\circ + 4.5a$

and if covered a distance 8 m during fifth and sixth seconds Then: $\frac{8}{2} = V_\circ + 5a$

and if covered a distance 18 m during Seventh, eighth and ninth seconds Then: $\frac{18}{3} = V_\circ + 7.5a$

(10) The motion is accelerated (velovity increasing) if the acceleration and velocity have the same direction ($av > 0$)

(11) The motion is decelerated (velocity decreasing) if the acceleration and velocity in opposite direction ($av < 0$)

Definition of the acceleration vector $\vec{A}$

It is the change of the velocity vector in the unit of time.

$$\vec{a} = \frac{\vec{V}_2 - \vec{V}_1}{\vec{t}_2 - \vec{t}_1}$$

($\vec{t}_2$, $\vec{t}_1$) are two successive moments

($\vec{V}_2$, $\vec{V}_1$) the velocity vector of the partical at respectively

Units of measuring acceleration

$(cm/\sec)/\sec \quad i.e \quad cm/\sec^2$

or $(m/\sec)/\sec \quad i.e \quad m/\sec^2$

$(1\ km./h)/\sec = \frac{1000\ m}{3600\ \text{"sec} \times \text{sec"}} = \frac{5}{18} m/\sec^2$

$(1\ km./h)/\sec = \frac{1000 \times 100\ cm}{3600\ \text{"sec} \times \text{sec"}} = \frac{250}{9} cm/\sec^2$

Study Carefully these questions

1 A prtical started its motion in a constant direction with initial velocitys 10 cm./sec and with uniform acceleration 4 cm./sec² in the same direction of the initial velocity. Find :
(1) its velocity at the end of one minute
(2) the displacement coveredduring this minute

$\mathcal{V}_\circ = 10 cm/\sec$, $a = 4 cm/\sec^2$, $t = 60 \sec$

(1) $V = \mathcal{V}_\circ + at = 10 + 4\times 60 = 250 cm./\sec$

(2) $S = \mathcal{V}_\circ t + \frac{1}{2}at^2 = 10\times 60 + \frac{1}{2}\times 4\times 60 = 600 + 7200 = 7800 cm.$

2 The velocity of a car uniformly decreased from 66 km./h., to 12 km./h., after it covered 585 m. Find the time taken by the car to cover the displacement and the distance which will be covered after wards to come to rest.

$\mathcal{V}_\circ = 66\times\frac{5}{18} = \frac{55}{3}\ m/\sec,\quad V = 12\times\frac{5}{18} = \frac{10}{3}\ m/\sec^2 \quad , \quad S = 585\ \sec$

$V^2 = \mathcal{V}_\circ^2 + 2aS \quad \Rightarrow \therefore (\frac{10}{3})^2 = (55)^2 + 2\times a\times 585$

$a = \frac{-5}{18}\ m./\sec^2$

$\because V = \mathcal{V}_\circ + at \quad \Rightarrow \therefore \frac{10}{3} = \frac{55}{3} + \frac{-5}{18}\times t$

$\therefore t = 54\ \sec$

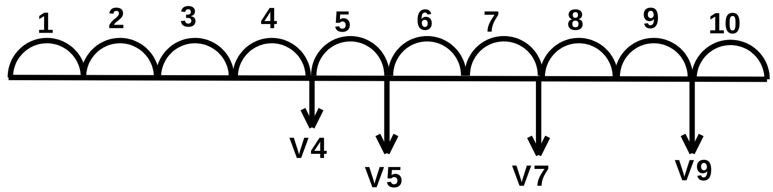
$V^2 = U^2 + 2AS \quad \Rightarrow \therefore 0 = (\frac{10}{3})^2 + 2\times A\times\frac{5}{18}\times S$

$\Rightarrow \therefore S = 20\ m$

3 A partical starts its motion in a constant direction at 10 cm/sec and uniform acceleration of 4 cm/sec² in the same direction of its velocity. Calculate :
(1) the distance covered by the partical in the fifth second only
(2) the distance covered by the partical in the 8th and 9th second together

$\mathcal{V}_\circ = 10\ cm/\sec \quad , \quad a = 4\ cm/\sec^2$

First : $S = \mathcal{V}_\circ t + \frac{1}{2}at$



$S_5 = 10\times5 + \frac{1}{2}\times4\times5^2 = 100\ cm$

$S_4 = 10\times5 + \frac{1}{2}\times4\times4^2 = 172\ cm$

S in the fifth second only $= S_5 - S_4 = 100 - 72 = 28\ cm$

second : $S = \mathcal{V}_\circ t + \frac{1}{2}at$

$S_7 = 10\times7 + \frac{1}{2}\times4\times7^2 = 168\ cm$

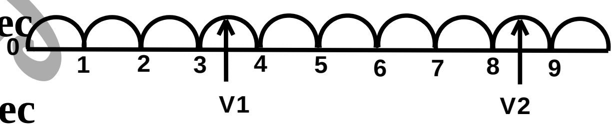
$S_9 = 10\times9 + \frac{1}{2}\times4\times9^2 = 252\ cm$

S in the 8^{th} and 9^{th} seconds $= S_9 - S_7 = 252 - 168 = 84\ cm$

4 *A body moves with uniform acceleration in a straight line. if it covered 26 m. during the 4^{th} second of its motion, 56 m., during the 9^{th} second. Calculate the acceleration by which it moves and its initial velocity*

Solution $\because V = \frac{S}{t}$

$V_1 = \frac{26}{1} = 26m/\sec \quad \Rightarrow \quad t_1 = 3.5\ \sec$



$V_2 = \frac{56}{1} = 56m/\sec \quad \Rightarrow \quad t_2 = 8.5\ \sec$

$a = \frac{V_2 - V_1}{t_2 - t_1} = \frac{56-26}{8.5-3.5} = \frac{30}{5} = 6\ m/\sec^2$

$V_1 = \mathcal{V}_\circ + at \quad \Rightarrow \quad \therefore\ 26 = \mathcal{V}_\circ + 6\times3.5$

$\mathcal{V}_\circ = 5\ m/\sec$

5 *A particle moved in a constant direction, it covered 18 m. during the first three seconds of its motion. 12 m. during the fifth second , 20 m. in the 9^{th} sec. prove that the particle was moving with uniform acceleration. calculate its velocity at the begining of motion*

during the first 3 seconds $= 18m.$

$S = \mathcal{V}_\circ t + \frac{1}{2}t^2$

$18 = \mathcal{V}_\circ \times 3 + \frac{1}{2}\times A\times3^2 \quad \Rightarrow \boxed{18 = 3\mathcal{V}_\circ + \frac{9}{2}a} \rightarrow (1)$

$\underline{during\ the\ fifth\ second\ = S_5 - S_4 = 12m.}$

$$S_5 = \mathcal{V}_{\circ} \times 5 + \frac{1}{2} \times a \times 5^2 \qquad \Rightarrow S_5 = 5\mathcal{V}_{\circ} + \frac{25}{2}a$$

$$S_4 = \mathcal{V}_{\circ} \times 4 + \frac{1}{2} \times a \times 4^2 \qquad \Rightarrow S_5 = 4\mathcal{V}_{\circ} + \frac{16}{2}a$$

$$S_5 - S_4 = 12m. \quad \Rightarrow \left(5\mathcal{V}_{\circ} + \frac{25}{2}a\right) - \left(4\mathcal{V}_{\circ} + \frac{16}{2}a\right) = 12 \qquad \boxed{12 = \mathcal{V}_{\circ} + \frac{9}{2}a} \rightarrow (2)$$

from (1) & (2) we get $\mathcal{V}_{\circ} = 3\ m./\sec$, $a = 2\ m./\sec^2$

$\underline{during\ the\ 9^{th}\ second\ = S_9 - S_8 = 20m.}$

$$S_9 = \mathcal{V}_{\circ} \times 9 + \frac{1}{2} \times a \times 9^2 \qquad \Rightarrow S_9 = 9\mathcal{V}_{\circ} + 32a$$

$$S_8 = \mathcal{V}_{\circ} \times 8 + \frac{1}{2} \times a \times 8^2 \qquad \Rightarrow S_8 = 8\mathcal{V}_{\circ} + \frac{64}{2}a$$

$$S_9 - S_8 = 20m. \qquad \Rightarrow \left(9U + \frac{81}{2}A\right) - \left(9U + \frac{81}{2}A\right) = 20$$

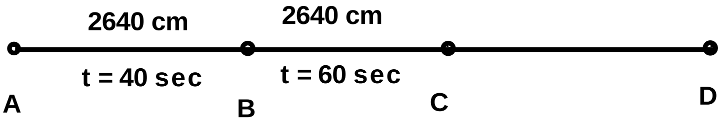
$$\therefore 20 = \mathcal{V}_{\circ} + \frac{17}{2}a \qquad \Rightarrow 20 = 3 + \frac{17}{2}a \qquad \Rightarrow a = 2m./\sec^2$$

$\therefore$ *the particle is moving with uniform acceleration.*

6 *A , B , C and D are four points on the same straight line such that AB = BC = 2640 cm., a body is moving from point A with uniform acceleration, it covered the distance AB in 40 sec., BC in 60 sec., In how many seconds it coveres the distance $\overline{CD}$, what is the length of $\overline{CD}$ if the body came to rest at D ?*

$S = \mathcal{V}_{\circ}t + at^2$

$$\therefore 2640 = 40\mathcal{V}_{\circ} + \frac{1}{2}(40)^2 a = 40\mathcal{V}_{\circ} + 800a$$



$$\therefore \boxed{66 = \mathcal{V}_{\circ} + 20a} \rightarrow (1)$$

$$\because 5280 = 100\mathcal{V}_{\circ} + \frac{1}{2} \times (100)^2 a = 100\mathcal{V}_{\circ} + 5000a$$

$$\boxed{52.8 = \mathcal{V}_{\circ} + 50a} \rightarrow (2)$$

from (1) & (2) we get $\mathcal{V}_{\circ} = 74.8\ m./\sec$, $a = -0.44\ m./\sec^2$

$$V = \mathcal{V}_{\circ} + at = 74.8 + (-0.44) \times 100 = 30.8\ cm./\sec$$

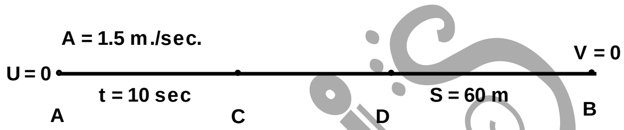
$$0 = 30.8 - 0.44 \times t_{CD} \qquad \Rightarrow t_{CD} = 70\sec.$$

$$S_{CD} = 30.8 \times 70 - \frac{1}{2} \times 0.44 \times (70)^2 \qquad \Rightarrow S_{CD} = 1078\ \sec.$$

7 *A train moves between two stations of 700 m., apart with acceleration of 1.5 m/esc². it started its motion from rest at the 1st station for 10 sec. then with uniform velocity for sometimes, then it covred the last 60 m. with uniform retardation until it came to rest at the next station. Find the time taken to cover the distance between the two stations.*

$V = V_\circ + at = 0 + \frac{3}{2} \times 10 = 15\ m/\sec$

$S = V_\circ t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{3}{2} \times 100 = 75m$



$CD = 700 - (60 + 75) = 565m$

but $V_B = V_{\circ D} + at = 15 + a^\backslash \times t = 0 \qquad \therefore \boxed{a^\backslash t = -15} \rightarrow (1)$

but $V^2_B = V^2_{\circ D} + 2a^\backslash S = (15)^2 + a^\backslash \times 60 = 0$

$\therefore \boxed{a^\backslash = -1.875\ m/\sec^2} \rightarrow (2)$ *from (1) & (2)*

$t_{BD} = 8\ \sec \quad , \quad t_{CD} = \frac{565}{15} = 37\frac{2}{3}\ \sec \qquad \therefore t = 10 + 8 + 37\frac{2}{3} = \frac{167}{3}\ \sec.$

8 *A train started its motion from a station A with uniform acceleration 15 cm./sec². it reached its maximum velovity after 2 minutes and moved with this velocity for $11\frac{2}{15}$ minutes, afterwards the brakes are used and it continued its motoin with a uniform retardation 150 cm/sec² until it came to rest at B. Find the distance AB and its average velocity to cut it*

about $\overrightarrow{AC}$, $\quad V_{\circ A} = 0 \quad , \quad a = 15\ cm/\sec^2 \quad , \quad t = 2\min = 120\sec$

$V = V_\circ + at = 0 + 120 \times 15 = 1800\ cm/\sec$

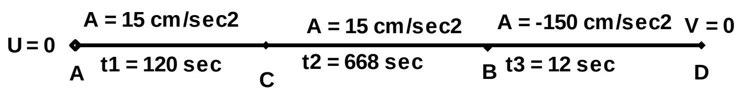
about $\overrightarrow{CB}$, $\quad V_{\circ C} = 1800\ , a = 15\ cm/\sec^2,\ t = 2\min = 11\frac{2}{15} \times 60 = 668\sec$

about $\overrightarrow{DB}$, $\quad V_{\circ C} = 1800\ cm/\sec \quad , \quad V = 0 \quad , \quad a = -150\ cm/\sec^2$

$V = V_\circ + at = 1800 - 150t \quad \Rightarrow \therefore t = 12\sec$

$S_{AC} = V_\circ t + \frac{1}{2}At^2 = 0 + \frac{1}{2} \times 15 \times (120)^2 = 108000\ cm$

$S_{CD} = Vt = 1800 \times 668 = 1202400\ cm$

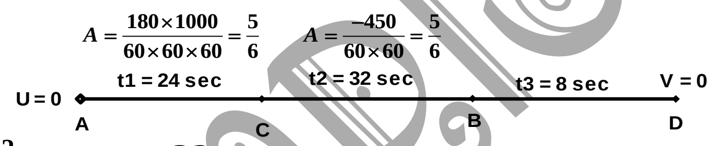


$S_{DB} = V_\circ t + \frac{1}{2}at^2 = 1800 \times 12 - \frac{1}{2} \times 150 \times (12)^2 = 10800\ cm$

$S_{AB} = 108000 + 1202400 + 10800 = 1321200\ cm$

average velocity $= \frac{\text{total distance}}{\text{total time}} = \frac{1321200}{(120 + 668 + 12)} = 1651.5\ cm/\sec$

9 *A car started its motion from rest with uniform acceleration 180 km/h² for every miniute. after 24 seconds the acceleration is stoped and the velocity decreased uniformly by means of friction and air resistance at a rate of 450 m/h/sec and after 32 seconds the brakes are used to stop it in 8 seconds. find the distance covered by the car.*



about $\overrightarrow{AC}$, $\mathcal{V}_{\circ A}=0$, $a=\frac{5}{6}\ m/\sec^2$, $t=24\sec$

$S=\mathcal{V}_{\circ}t+\frac{1}{2}at^2=0+\frac{1}{2}\times\frac{5}{6}\times(24)^2=\boxed{240\ m}$

$V=\mathcal{V}_{\circ}+at=0+\frac{5}{6}\times24=20\ m/\sec$

about $\overrightarrow{CD}$, $\mathcal{V}_{\circ C}=20$, $a=\frac{-1}{8}\ m/\sec^2$, $t=32\sec$

$S=\mathcal{V}_{\circ}t+\frac{1}{2}At^2=20\times32-\frac{1}{2}\times\frac{1}{8}\times(32)^2=\boxed{576\ m}$

$V=U+At=20-\frac{1}{8}\times32=16\ m/\sec$

about $\overrightarrow{DB}$, $\mathcal{V}_{\circ D}=16$, $\mathcal{V}_{\circ B}=0$, $t=8\sec$

$V_B=\mathcal{V}_{\circ D}+at \quad \Rightarrow\therefore\ 0=16+8A \quad \Rightarrow\therefore\ a=-2m/\sec^2$

$S=\mathcal{V}_{\circ}t+\frac{1}{2}at^2=16\times8-\frac{1}{2}\times2\times(8)^2=\boxed{64\ m}$

$S_{AB}=240+576+64=880\ m$

10 *A car moving with uniform velovity 72 km/h passed by a police car at rest, the police car started motion following it after 10 seconds with uniform acceleration a distance 100 m. until the velocity be came 90 km/h, then it moved with this velocity until it overtook the first car, find the time taken during this procassfrom the bigining of motion of the police car and the distance covered by this car*

$V_P=\frac{90\times1000}{60\times60}=25m/\sec$, $V_{car}=\frac{72\times1000}{60\times60}=20\ m/\sec$

about the police car,

$V^2=\mathcal{V}_{\circ}^2+2aS$

$(25)^2=0^2+2a\times100 \qquad \Rightarrow\therefore a=3.125\ m/\sec^2$

$V=\mathcal{V}_{\circ}+at \qquad \therefore 25=0+3.125\times t \qquad \Rightarrow\therefore\ t=8\sec$

about the car,

$S\ after\ 10\ \sec\ =V\times T=2010=200\ m$

at the point of meeting, the two covered distance are bequal

$\therefore\ 100+25(t-8) = 200+20t$

$100+25t-200 = 200+20t$

$5t = 300 \qquad t = 60 \text{ sec.} = 1 \text{ min}$

The total covered distance $= 200 + 20\times 60 = 1400\ m.$

Homework

1. Complete the following:

   a. A particle moves in a straight line from rest with uniform acceleration of magnitude $4 m/sec^2$ so, its velocity after 6 seconds from starting the motion = ... m/sec.

   b. The distance that the particle covered in a constant direction from rest with acceleration. of magnitude $5 cm/sec^2$ during a time of magnitude 4 seconds = ... cm.

   c. The average velocity for a particle moving with initial velocity v_0 and a uniform acceleration (a) through the sixth second from its motion = ...

   d. The average velocity for a particle moving with initial velocity V_0 and a uniform acceleration (a) through the seconds 7^{th}, 8^{th} and 9^{th} from the starting of the motion = ...

   e. A particle moves from the rest in a straight line with uniform acceleration. So, it covered 24 meters in the first four seconds from its motion , then the magnitude of its acceleration = ...

   f. A particle started its motion from rest in a straight line with uniform acceleration of magnitude $2 cm/sec^2$ so, it covered a distance 25 cm ,then its velocity at the end of this distance = ... cm/sec.

2. A car moves from rest with acceleration of magnitude $4 m/sec^2$. What is the distance that the car covered when its velocity became 24 m/sec?

3. A racing car moves in the track with velocity 44 m/sec then its velocity decreases with a constant rate until it becomes 22 m/sec through 11 seconds. Find the distance that the car covered through that time .

4. A particle moves in a straight line with a uniform acceleration so its velocity increased from 15 m/sec into 25 m/sec. after covering 125 meter .Find the time takes for that .

5. A cyclist moves with a uniform acceleration until its velocity became 7.5 m/sec through 4.5 seconds. If the displacement of the bicycle through the accelerating interval equals 19 meters, find the initial velocity for the bicycle.

6. Karim practices on riding the bicycle. His father pushes him to gain a constant acceleration of magnitude $\frac{1}{2} m / sec^2$ for 6 seconds and after that Karim rides the bicycle alone with the velocity gained for another 6 seconds before he falls on the ground. Find the distance that Karim will cover.

7. A cyclist descends from the top of a hill with a constant acceleration of magnitude $2 m/sec^2$. When he reaches the base of the hill, his velocity reaches 18 m/sec. then he uses the brakes to preserve this velocity for one minute. Find the total distance that the cyclist covered.

8. A car driver moves with a constant velocity of magnitude 24 m/sec. He suddenly saw a child passing the road. If the required time for the brakes to respond is $\frac{1}{2}$ sec then it moves with a uniform deceleration of magnitude $9.6 m/sec^2$ until it stopped. Find the total distance covered by the car before it stops.

(9) A body started its motion from rest in a horizontal straight line with uniform acceleration of magnitude 4 cm/sec^2 for 30 seconds ,then it moves with the velocity it gained for another 40 seconds. Find the magnitude of its average velocity.

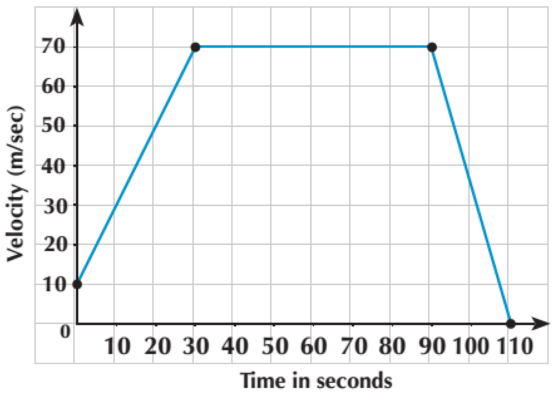
(10) A body moves in a straight line with uniform acceleration on a smooth horizontal plane till it covered 26 meters through the 4th second from starting the motion and 56 meters through the 9th second only , Find its initial velocity and the magnitude of its acceleration.

(11) x, y are two points on a horizontal straight road. The car (A) started the motion from x towards y starting from rest and with uniform acceleration 10 m/sec^2 and at the same moment another car (B) moves from y towards x with uniform velocity of magnitude 54 km/hr , if the relative velocity for the car (A) with respect to the car (B) at the moment of their meeting equals 162 km/hr ,find the time taken by each one of the two cars from the moment of their motion together until the moment of their meeting.

Activity

(12) The opposite figure represents the curve (the velocity - the time) for a body started the motion with an initial velocity of magnitude 10 m/sec and until it came to rest after a time of magnitude 110 second. Find:

- **a** The acceleration.
- **b** The magnitude of the uniform deceleration for the body until it rests.
- **c** The total distance that the body moves.

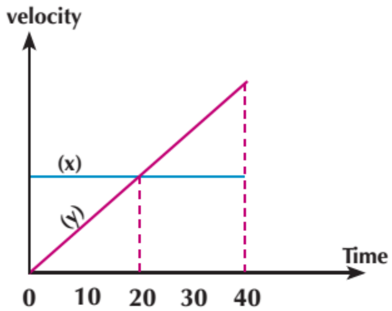


Creative thinking:

(13) A lift is at rest at the bottom of a mine. The lift rises a distance 540 cm with acceleration of magnitude 120 cm/ sec^2, then it moves with uniform velocity for a distance 360 cm then with a uniform deceleration a distance 720 cm until it rests at the nozzle of the mine. Calculate the time that the lift takes in ascending from the bottom of the mine to its nozzle.

Creative thinking:

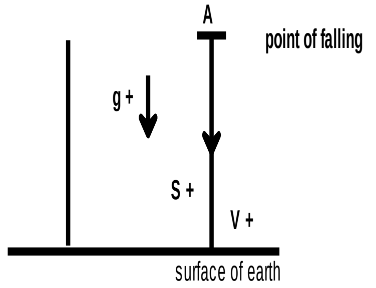
(14) The opposite figure represents the curve (velocity – distance) for two cars x and y find the time taken by the two cars till they met.(Explain your answer)



Vertical motion under gravity

First :

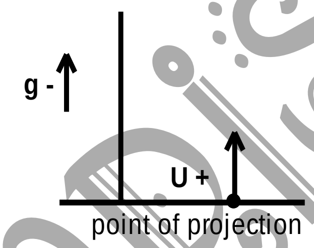
If the body is falling or projected downwards



Second :

If the body is projected vertically upwards

g is negative , $\mathcal{V}_\circ$ is positive



Laws of vertical motion

First (falling or projected downwards):

(1) $V = \mathcal{V}_\circ + g\,t$

(2) $S = \mathcal{V}_\circ\,t + \frac{1}{2}\,g\,t^2$

(3) $V^2 = \mathcal{V}_\circ^2 + 2\,g\,S$

Second (projected vertically upwards):

(1) $V = \mathcal{V}_\circ - g\,t$

(2) $S = \mathcal{V}_\circ\,t - \frac{1}{2}\,g\,t^2$

(3) $V^2 = \mathcal{V}_\circ^2 - 2\,g\,S$

$\|\vec{g}\| = 9.8\ m/\sec^2 = 980\ cm/\sec^2$

Important remarks to solve problems

If a body is projected vertically upwards, then :

(1) The time of maximum height $= \frac{\mathcal{V}_\circ}{g}$

(2) To Find the maximum height $= \frac{\mathcal{V}_\circ^2}{2g}$

(3) Time taken to reach the max. height = time of moving downwards to reach the same point of project

(4) The velocity of projection from the point of projection upwards = the velocity with which the body reaches the point of projection but in opposite direction (sign $\pm \leftrightarrow \mp$*)*

(5) The displacement (S) is positive above the point of projection and negative under the point of projection and equal zero at the point of projection

1 *A stone is projected in a well with velocity 2 m/sec, it reached its bottom after 3 sec, find the velocity of the stone which it strikes the base of the well and find the depth of the well.*

$U = 2 \ m/\sec \quad , \quad t = 3 \ \sec$

(1) $V = U + gt = 2 + 9.8 \times 3 = 31.4 \ cm./\sec$

$S = Ut + \frac{1}{2}gt^2 = 2 \times 3 + \frac{1}{2} \times 9.8 \times 3^2 = 50.10 \ cm.$ *(depth of the well)*

2 *A particle is projected vertically upwards with velocity 28 m/ sec Find the time elapsed to reach the maximum height and the distance attained*

$V = U - gt \quad \Rightarrow \therefore 0 = 28 - 9.8t \Rightarrow \quad t = \frac{20}{7} \ \sec$

$V^2 = U^2 - 2gS \quad \Rightarrow \therefore 0^2 = (28)^2 - 2 \times 9.8 \times \frac{5}{18} \times S$

$\Rightarrow \therefore S = 40m$

$another \ solution \Rightarrow \quad t = \frac{U}{g} = \frac{28}{9.8} = \frac{20}{7} \sec$

$S = \frac{U^2}{2g} = \frac{(28)^2}{2 \times 9.8} = 40m.$

3 *A body is projected vertically with a velocity of 14.7 m/sec. From a point on the ground. calculate the velocity of the body when it becomes at a height 9.8 m. from the ground. Explain the meaning of the two answers.*

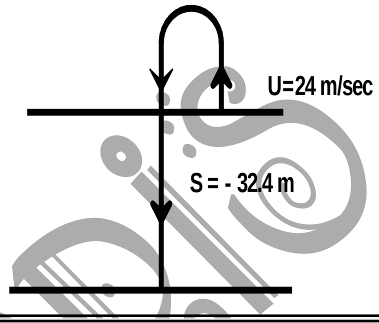
$V^2 = U^2 - 2gS = (14.7)^2 - 2 \times 9.8 \times 9.8 = 24.01$ *by* $\sqrt{\ }$

$V = \pm 4.9m/\sec$

V=4.9 m/sec when the body is moving upwards

V= -4.9 m/sec when the body is moving downwards

4 *A particle is projected vertically upwards with velocity 24 m/ se Find the time taken until the body reaches a point 32.4 m below the point of projection*



$$S = Ut + \frac{1}{2}gt^2 \qquad \Rightarrow \therefore -32.4 = 24t - \frac{1}{2}\times 9.8t^2$$

$$4.9t^2 - 24t - 32.4 = 0$$

$$(t-6)(4.9t+5.4) = 0$$

$$t = 6\text{sec} \quad or \quad t = \frac{-54}{49} \quad (refused)$$

5 *A body fell vertically downward. find its velocity after 4 sec. and the time elapsed to cover 68.6 m.*

$$V = U + gt \qquad \Rightarrow \therefore V = 0 + 4\times 9.8 = 39.2 m/\text{sec}$$

$$S = Ut + \frac{1}{2}gt^2 \qquad \Rightarrow \therefore 68.6 = 0 + \frac{1}{2}\times 9.8\times t^2$$

$$\therefore t^2 = \sqrt{14} = 3.74\text{sec}.$$

6 *A stone is projected in a well with velocity 4 m/sec. If reached its bottom after 2 sec. find the depth of the well and the stone velocity when it strikes its bottom*

$$V = U + 2gt = 4 + 9.8\times 2 = 24.01 \quad by\ \sqrt{}$$

$$V = 24.6\ m/\text{sec}$$

$$S = Ut + \frac{1}{2}gt^2 = 4\times 2 + \frac{1}{2}\times 9.8\times 2^2 = 27.6\ m$$

7 *A particle is projected vertically upward with velocity 19.6 m/sec. Find the time taken to reach the maximum height and this maximum height*

$$t = \frac{U}{g} = \frac{19.6}{9.8} = 2\text{sec}.$$

$$\max.height = \frac{U^2}{2g} = \frac{(19.6)^2}{2\times 9.8} = 19.6\ m$$

8 *A particle is projected vertically upwards with velocity 24.5m/sec., after how many second it comes back the point of projection*

$V = U - gt \quad \Rightarrow \therefore 0 = 24.5 - 9.8t$

$t = 2.5$

the total time = 2×2.5 = 5 sec.

9 *A particle moves vertically with velocity 19.6 m/sec. from a point on the ground. find when it becomes at 14.7 m. above the ground and its velocity, then Explain meaning of the two answers*

$S = Ut - \frac{1}{2}gt^2 \quad \Rightarrow \therefore 14.7 = 19.6t - \frac{1}{2}\times 9.8\times t^2$

$4.9t^2 - 19.6t + 14.7 = 0 \quad \Rightarrow \therefore t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 1 \quad \Rightarrow \quad t = 3 \quad , \quad t = 1\text{sec}$

this particle becomes at this height upward after 1 sec.
and becomes at this height downward after 3 sec.

$V = u - gt = 19.6 - 9.8\times 1 = 9.8 \ m/\sec \ (upward)$

$V = u - gt = 19.6 - 9.8\times 3 = -9.8 \ m/\sec \ (downward)$

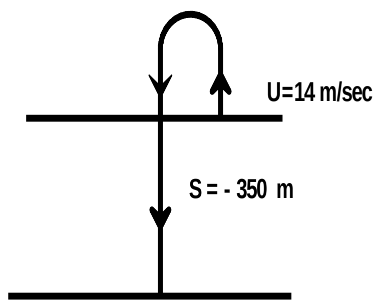
10 *A particle is projected upwards with velocity 14 m/sec. find the time taken to reach a point 350 m below the point of projection*

$S = Ut - \frac{1}{2}gt^2 \Rightarrow \therefore -350 = 14t - \frac{1}{2}\times 9.8t^2$

$4.9t^2 - 14t - 350 = 0$

$(4.9t + 35)(t - 10) = 0$

$t = 10 \quad or \quad t = \frac{-35}{4.9} \quad (refused)$



11 *A stone is projected vertically upward with velocity 19.6 m/sec. from the top of a tower 156.8 m. Find when it reaches the ground and its velocity at this instant.*

$U = 19.6\ m/sec.$, $S = -156.8\ m.$

$S = Ut - \frac{1}{2}gt^2 \qquad \Rightarrow \therefore -156.8 = 19.6t - \frac{1}{2}\times 9.8\times t^2$

$t^2 - 4t - 32 = 0$

$(t-8)(t+4) = 0 \qquad \Rightarrow \therefore t = 8\sec.$

$V = U - gt \qquad \Rightarrow \therefore V = 19.6 - 9.8\times 8 = -58.8 m/\sec$

12 *A small ball is projected vertically upward from the window of a house, the ball was observed moving dowround against the window after 4 sec. then reached the ground after 5 sec. from the instant of projection. Find the height of the window in meters.*

$V = U - gt \qquad \Rightarrow \therefore 0 = U - 9.8\times 2$

$U = 19.6 m/\sec$

$S = Ut + \frac{1}{2}gt^2 \qquad \Rightarrow \therefore S = 19.6\times 1 + \frac{1}{2}\times 9.8\times 1^2$

$S = 24.5m$

Homework

1. A child throws a ball from a window that rises 3.6 m from the pavement. What will be its velocity at the moment of contact with the pavement?
2. A ball fell vertically downwards. What is its velocity after 6 seconds from the moment of its falling?
3. A body fell vertically downwards from height 490 m from the surface of the ground find:
   - a Time of reaching the ground surface.
   - b Its velocity after 5 seconds from starting the motion.
4. A rubber ball fell from a height of 10 meters so it hit the ground and rebounded vertically upward a distance $2\frac{1}{2}$ meters. Calculate the velocity of the ball just after and before hitting the ground.
5. A student practices on kicking football vertically upwards in air, then the ball returns due to the impact of every kick. So, it hits his foot. If the ball takes from the moment of its kicking until colliding with his foot 0.3 seconds.
   - a Find the initial velocity.
   - b The height that the ball reaches after the student kicked it.

6. A body is projected vertically upwards from the top of a hill of height 9.8 meters with velocity 4.9 m/sec Find:

   a. Velocity of the body at the moment of reaching the bottom of the hill.

   b. The time taken to reach the bottom of the hill.

7. A stone is projected in a well with velocity 4 m/sec vertically downwards so, it reached the bottom of the well after 2 seconds. Find:

   a. The depth of the well.

   b. Velocity of the stone when it collides with bottom of the well.

8. A particle is projected vertically upwards with velocity 14 m/sec from a point at height 350m from ground surface. Find the time that the particle takes to reach the ground surface.

9. A ball is projected upwards from a window. So, it reaches it after 4 seconds from the moment of the projection and it reached the ground surface after 5 seconds from the moment of the projection. Find:

   a. Velocity of the ball's projection.

   b. Maximum height that the ball reached from the point of the projection.

   c. The height of the window from the ground surface.

10. A body is projected vertically upwards from the top of a tower its height 80.5 meters with velocity 8.4 m/sec. Find:

    a. The maximum height that the body reaches from the point of the projection.

    b. The time that the body takes while descending until its velocity become 11.2 m/sec.

    c. The time taken by the body to reach the projection point.

    d. The time taken by the body to reach the ground surface.

11. A ball is projected from the top of the hill of height 140 m vertically upwards; it is found that it covered in the third second a distance 10.5 meters. Find:

    a. The velocity that the ball is projected with.

    b. The maximum height that the ball reached.

    c. The time that the ball takes to reach the ground surface.

Creative thinking:

12. A body fall from a height of 60 meters from the ground surface and at the same moment, another body is projected vertically upwards from the ground surface with velocity 20 m/sec. The two bodies meet after a time interval. Find this time, and then find the distance that the two bodies covered during this time interval. Mention whither the two bodies met each other moving in two opposite directions or in the same directions?

Universal gravitational law

The forces of attraction between two bodies are directly proportional to the product of their masses and indirectly proportional with the square of distance between their centers

Let m_1 , m_2 are the masses of two bodies and the distance between their centers s: Then

$$F \alpha \frac{m_1 \times m_2}{s^2} \quad \Rightarrow \therefore F = G \times \frac{m_1 \times m_2}{s^2}$$

M measured in kg. , S in meter , f in newton

Example (1) Calculate the gravitational force between two bodies their masses are 10 kg, 50 kg and distance between their centers 0.5 m. known that the universal gravitational constant equals 6.67×10^{-11} newton .m^2/kg^2

Solution

$$F = G \times \frac{m_1 \times m_2}{s^2} \quad \Rightarrow \therefore F = 6.67 \times 10^{-11} \times \frac{10 \times 50}{(0.5)^2} = 1.334 \times 10^{-8}$$

Example (2) Calculate the gravitational force between two planets the mass of the first 2×10^{21} ton, mass of the other 4×10^{25} ton and the distance between their centers 2×10^{6} km

Solution

$$F = G \times \frac{m_1 \times m_2}{s^2}$$

$$F = 6.67 \times 10^{-11} \times \frac{2 \times 10^{24} \times 4 \times 10^{28}}{(2 \times 10^{9})^2} = 1.334 \times 10^{-8}$$

$$F = 6.67 \times 10^{-11} \times \frac{2 \times 10^{52}}{4 \times 10^{18}} = 6.67 \times 10^{-11} \times 2 \times 10^{34}$$

$$F = 1.334 \times 10^{24} \ newton$$

Example (3) A piece of iron is put at a distance of 50 cm from another piece of Nickel of mass 25 kg then the gravitational force between them became $6x10^{-8}$ N. What is the mass of the piece of iron approximated to the nearest integer number ?

Solution

$$F = G\times\frac{m_1\times m_2}{s^2}$$

$$6\times10^{-8} = 6.67\times10^{-11}\times\frac{m_1\times25}{(0.5)^2}$$

$$m_1 = \frac{0.25\times6\times10^{-8}}{6.67\times10^{-11}\times25} = 9\text{ kg.}$$

Example (4) A satellite of mass 1500 kg revolves at a height of 540 km from the Earth's surface whose mass is $6x10^{24}$ kg and radius is 6360 km. Find the Earth's gravitational force on the moon known that the universal gravitational constant equals $6.67x10^{-11}$ newton.m^2/kg^2

Solution

$$F = G\times\frac{m_1\times m_2}{s^2}$$

$$F = 6.67\times10^{-11}\times\frac{1500\times6\times10^{24}}{(540000+6360000)^2} = 1260807\ \textit{newton}$$

Example (5) If the attraction force between the Earth and the Moon is $3011x10^{24}$ newton and the mass of the Earth $6x10^{24}$ kg, the mass of the Moon $7x10^{22}$ kg. Find the distance between their centers.

Solution

$$F = G\times\frac{m_1\times m_2}{s^2}$$

$$3011\times10^4 = 6.67\times10^{-11}\times\frac{7\times10^{22}\times6\times10^{24}}{S^2}$$

$$S = \sqrt{\frac{7\times10^{22}\times6\times10^{24}\times6.67\times10^{-11}}{3011\times10^4}} = 3\times10^6\ \textit{meters}$$

Example (6) Calculate the mass of the Earth in kg if known that its radius length 6360 km and gravitational constant is $6.67x10^{-11}$ and the Earth's acceleration due to gravity is $9.8\ m/s^2$

Solution

Let a body of mass m_1 put on the surface of the Earth and m_2 is the mass of the Earth

The forces of attraction between the body and the Earth is the weight of the body itself = $m_1 \times g$

$$F = G \times \frac{m_1 \times m_2}{s^2}$$

$$\cancel{m_1} \times g = 6.67 \times 10^{-11} \times \frac{\cancel{m_1} \times m_2}{(6360000)^2}$$

$$9.8 = 6.67 \times 10^{-11} \times \frac{\cancel{m_1} \times m_2}{4.045 \times 10^{24}} \Rightarrow \therefore m_2 = 5.9 \times 10^{24} kg$$

Comparing the accelerations due to gravities on the surfaces of two planets:

If g_1, g_2 the acceleration due to gravity for each planet, m_1, m_2 their mass in kg, r_1, r_2 their radii in meters respectively, then from the previous

it is possible to deduce the following relation: $\frac{g_1}{g_2} = \frac{m_1}{m_2} \times \frac{r_2^2}{r_1^2}$

If the mass of the Earth is 81 times the mass of the moon and their diameter equal 12756 km, 3476 km respectively. If the acceleration due to gravity on the Earth equals $9.8\ m/sec^2$ what is the acceleration due to gravity on the moon's surface?

Solution

Let the mass of the moon is (m) kg ,then the mass of the Earth equals (81 m)

$r_1 = 6378$ km , $r_2 = 1738$ km , $g_1 = 9.8\ m/sec^2$, $g_2 =$ ؟

$\because \frac{g_1}{g_2} = \frac{m_1}{m_2} \times \frac{r_2^2}{r_1^2}$ $\quad\quad \therefore \frac{9.8}{r_2} = \frac{81\ m}{m} \times (\frac{1738}{6378})^2$

Simplifying: $\quad \therefore g\ moon \simeq 1.63\ m/sec^2$

Homework

1 Two identical balls each of mass 6.8 kg and the distance between their centers equals 21.8 cm. What is the gravitational force between them?

2 Calculate the gravitational force between two bodies of masses 10 kg , 15 kg and the distance between them is 2 meters.

3 A satellite of mass 2000 kg revolves at a height 440 km from the Earth's surface whose mass equals 6×10^{24} kg . Find the gravitational force of the Earth on the satellite knowing that the Earth's radius equals 6360 km.

4 If the Earth's acceleration due to gravity (g) is 10 m / sec^2 the Earth's radius equals 6.36×10^6 m. Calculate the Earth's mass.

5 Calculate the gravitational force between the Sun and the Earth if you know that the Earth moves in an elliptical orbit around the sun, the Earth's mass equals 6×10^{24} kg the sun's mass equals 9×10^{29} kg and the distance between their centers equals 1.5×10^{11} m.

6 If you know that the Earth's mass equals 5.97×10^{24} and its radius 6.34×10^6 m, the mass of the moon equals 7.36×10^{22} kg, find the radius of the moon if the gravity on the Earth's surface equals six times the gravity on the moon's surface.

7 If you know that the Earth's mass equals 6.06×10^{24} and its radius is 6.36×10^6 , find the intensity of the Earth's gravitational field.

8 A planet whose mass equals 3 times the Earth's mass, and its diameter equals 3 times the Earth's diameter. Calculate the ratio between the acceleration due to gravity on this planet and that on the Earth.

9 Find the universal gravitational force between two planets the mass of the first equals 2×10^{21} ton and the mass of the second equals 4×10^{25} ton and the distance between their centers equals 2×10^6 km.

10 A piece of iron is put at a distance of 50 cm from another piece of Nickel of mass 25 kg then the gravitational force between them became $10^{-8} \times 6$ N. What is the mass of the piece of iron approximated to the nearest integer number?

11 If a body of math m\ kg on a height s meter from the surface of the earth whose radius is r meter and mass m kg .find the value of the gravitational force that act on the body.

12 **Join with space:** an international space station with weight on the surface of the earth 421997.6 newton .find its weight when it became in an external orbit on a height 350 km from the surface of the earth known that the Earth's mass equals 6.37×10^3 and its radius is 5.6×10^{24} . (Hint : Force in Newton = Mass in kg × gravitational force of the earth 9.8 m/sec^2)

General Exercises (Unit two)

Complete the following:

1. If $\overrightarrow{V_A} = 7\ \overrightarrow{i}$, $\overrightarrow{V_B} = -3\ \overrightarrow{i}$ then $\overrightarrow{V_{AB}}$ = ...

2. If $\overrightarrow{V_{CD}} = 70\ \overrightarrow{n}$, $\overrightarrow{V_c} = 50\ \overrightarrow{n}$ then $\overrightarrow{V_D}$ = ...

3. If two cars A, B are moving with two velocities 65 km/h , 75 km/h ,then
   - a V_{AB} = ... if they are moving in the same direction.
   - b V_{AB} = ... if they are moving in opposite directions.

4. A car started its motion from rest with a uniform acceleration 20 cm/sec^2 for 10 seconds.
   - a The final velocity of the car = ... m/sec.
   - b The distance covered during this interval = ... m.

5. A body started its motion from rest with a uniform acceleration 72 km/hr and deceleration 2m/sec^2.
   - a The time required for the body to stop = ... sec.
   - b The distance covered during this interval = ... m.

6. A car used its brakes to stop in 10 seconds after covering a distance of 25 m.
   - a The acceleration of the body during using the brakes = ... m/sec^2.
   - b The velocity of the body at the beginning of using the brakes = ... m/sec.

7. body has fallen from the top of a vertical tower to reach the ground after 5 seconds:
   - a The velocity of the body when it reached the ground = ... m/sec.
   - b The height of the tower = ... m.

8. A body is projected vertically upwards from a point on the Earth's surface to return to it after 4 seconds:
   - a The velocity of projecting the body = ... m/sec.
   - b The maximum height reached by the body = ... m.

9. From the top of a tower of 20 m height a body is projected upwards with velocity 7m/sec:
   - a The velocity of reaching the ground = ... m/sec.
   - b The time interval to reach the ground = ... seconds.

10. A planet of mass equals 3 times the Earth's mass and its diameter equals 3 times the Earth's diameter, then the ratio between the acceleration due to gravity on this planet to the acceleration due to gravity on the Earth .... : ....

11. A body moves in a straight line a distance of 100 m with velocity 5 m/sec. Then it moves with a velocity of 8 m/sec. in the same direction for 10 seconds. Find the average velocity during the whole trip.

12) Two bodies A, B move in a straight line in the direction $\overrightarrow{BA}$ with two velocities 100 m/min, 120 km/hr. If the distance between them equals 30 km. find where and when they will meet?

13) A car (A) moving on a straight road has measured the relative velocity of another car (B) coming from the opposite direction to find it 130 km/hr. and when the car (A) doubled its velocity and measure of car B again , it found that it has become 180 km/hr. Find the actual velocities of the two cars.

14) A controlling speed car (Radar) moves on the highway (desert road) with a speed of 30 km/hr , it watched a truck coming from the opposite direction which seemed to be moving with velocity 110 km/h. Find the actual velocity of the truck.

15) A body has moved in a straight line with a velocity 7 m/sec.and with acceleration 4 m/sec^2. Find its speed and the distance cut by it in 6 sec.

16) A body started its motion from rest with uniform acceleration 20 km/sec^2 when its velocity became 8 m/sec it moved with deceleration until it stopped after 112 seconds from the start of the motion. Calculate the deceleration and the total distance covered by the body.

17) A body started its motion from rest to cover 150 m and when its velocity became 54 km/hr the acceleration stopped and it moved with its acquired velocity a distance of 300 m, then it moved with a uniform deceleration of value $\frac{3}{2}$ m/sec^2 till it stopped. Calculate the average velocity during the whole trip.

18) A body moves in a straight line with uniform acceleration to cover 52 meters in the first four seconds, then it cover a distance of 92 meters in the next four seconds. Calculate the acceleration of motion, the initial velocity and the distance covered during the first 10 seconds of its motion.

19) If $\overrightarrow{r}$ is the position vector of a body that moves in a straight line from point "O" which is determined by the relation $\overrightarrow{r} = (3n^2 - 3)\overrightarrow{n}$ find the displacement vector after 4 seconds from the starting of the motion.

20) A body has fallen from a high S above the Earth's surface to cover a distance of 34.3 m at the last second of its motion . Find:

a) The velocity of the body when it reaches the Earth's surface.

b) The height from which the body has fallen.

21) A body is projected vertically upwards with a velocity of 14 m/sec from a point at height 350 meters above the Earth's surface. Find:

a) The time taken by the body to reach the Earth's surface.

b) The total distance covered by the body to reach the Earth's surface.

22) A sphere of iron is put at a distance of 40 cm from another sphere of Nikcel of mass 50 kg such that the gravitational force between them equals 12×10^{-8} N, then what is the mass of the sphere of iron if you know that the universal gravitational constant equals 6.67×10^{-11} newton . m^2/kg^2

Chapter Four

The probability

Dif:1 The random experiment

It is an experiment which we know in advance all its possible outcomes but we cannot predict which of these outcomes will occure when the experiment. carried on

Dif:2 The Sample space

It is the set of all possible outcomes of the random experiment. and denoted by S.

Example.1

In the following exp. write down the sample space:

i) Tossing a coin one time.

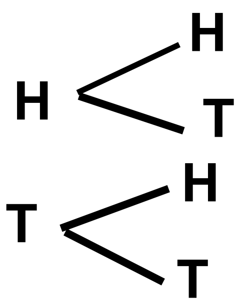
ii) Tossing a coin two times.

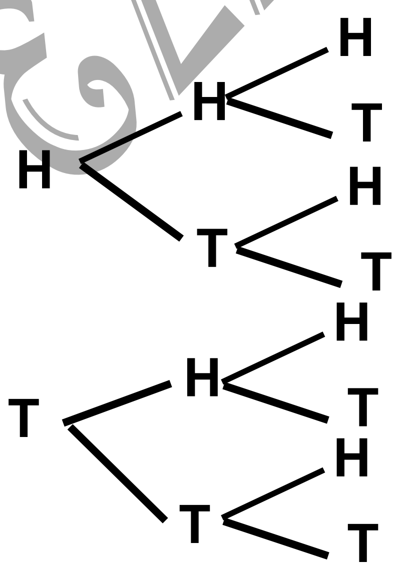
iii) Tossing a coin three times.

Solution

i) S = { (H,T) } , n(s) = 2^1 = 2

ii) S = { (H,H) , (H,T) , (T,T) , (T,H) } , n(s) = 2^2 = 4





iii) S = { (H,H,H) , (H,H,T) , (H,T,H) , (H,T,T),
(T,H,H) , (T,H,T) , (T,T,H) , (T,T,T) } , n(s) = 2^3 = 8

Example.2

In the following exp. write down the sample space:

i) Tossing a die one time.

ii) Tossing a die two times.

Solution

i) S = { 1 , 2 , 3 , 4 , 5 , 6 } , n(s) = 2^1= 2

ii)

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

, n(s) = 6^2 = 36

Types of events

1) The certain event

It is the event which must occure when we do the experiment

2) The impossible event ϕ

It is the event which is impossile to occure

3) The simple event

It is the set which contain one element

4) mutually exclusive

They cannot occure simultaneously

Example.3

In the following exp. write down the sample space:

i) Tossing a coin then a die one time.

Solution

i) S = { (H,1) , (H,2) , (H,3) , (H,4) , (H,5) , (H,6) ,
(T,1) , (T,2) , (T,3) , (T,4) , (T,5) , (T,6) }

Example.4

In the following exp. write down the sample space:
Selection of two cards one after the other with replacement from a box containing 4 cards numbered from 1 to 4

Solution **" with replacement"**

S = { (1,1) , (1,2) , (1,3) , (1,4) , (2,1) , (2,2) , (2,3) , (2,4) ,
(3,1) , (3,2) , (3,3) , (3,4) , (4,1) , (4,2) , (4,3) , (4,4) }

" without replacement"

S = { (1,2) , (1,3) , (1,4) , (2,1) , (2,3) , (2,4) ,
(3,1) , (3,2) , (3,4) , (4,1) , (4,2) , (4,3) }

Axioms of probability

(1) $P(A) = \dfrac{\text{The numbers of elements of A}}{\text{The number of elements of S}} \Rightarrow P(A) = \dfrac{n(A)}{n(S)}$

(2) $0 \le P(A) \le 1$

(3) $P(\varnothing) = 0$, $P(S) = 1$

Rule. 1

If A , B are two mutually exclusive events then $P(A \cap B) = 0$

Rule. 2

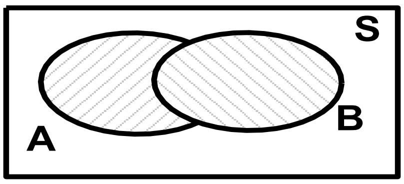
If $A \subset B$, then $P(A \cap B) = P(A)$, and $P(A \cup B) = P(B)$

Operations on events

(1) Occ. of A or B

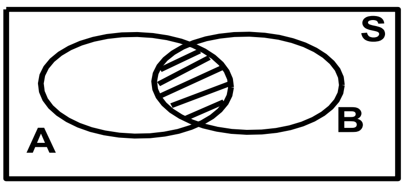
Occ. of one of them at least

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



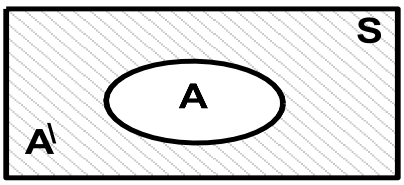
(2) Occ. of A and B together

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$



(3) Non occ. of A

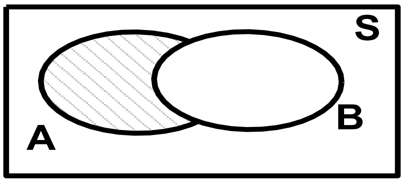
$P(A^{\backslash}) = 1 - P(A)$



(4) Occ. of A and non B

Occ. of A only

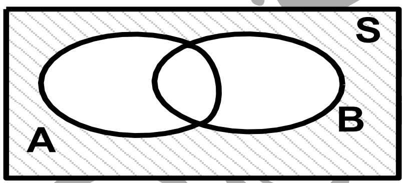
$$P(A-B) = P(A\cap B') = P(A) - P(A\cap B)$$



---

(5) Non occ. of neither A nor B

$$P(A'\cap B') = P(A\cup B)' = 1 - P(A\cup B)$$

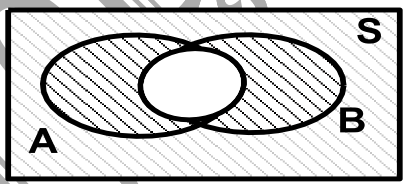


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(6) Non occ. of A and B together

at most one of them

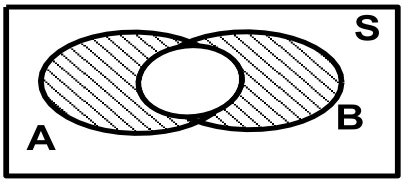
$$P(A'\cup B') = P(A\cap B)' = 1 - P(A\cap B)$$



---

(7) Occ. of only one of A or B

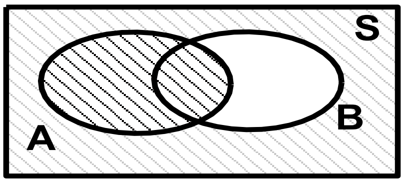
$$P[(A-B)\cup(B-A)] = p(A) + P(B) - 2P(A\cap B)$$



---

(8) Occ. of A or non occ. of B

$$P(A\cup B') = 1 - P(B-A)$$



---

(9) $P(A'-B') = P(B-A) = P(B) - P(A\cap B)$

---

(10) $P(A-B') = P(A\cap B)$

---

(11) $P(A'-B) = P(A'\cap B') = 1 - P(A\cup B)$

---

(12) $P(A-B)' = 1 - P(A-B)$

Example.1

One card is drawn randomly from a box containing 9 cards numbered from 1 to 9. describe the sample space of this exp. and then calculate the peob. of the following events:

A: The event of the occ. of an odd number on the selected card
B: The event of the occ. of a prime number on the selected card
C: The event of the occ. of A or B
D: The event of the occ. of A and B
E: The event of the occ. of only A
F: The event of non occ. of neither A nor B

Solution:

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 3, 5, 7, 9\}$ $P(A) = \frac{5}{9}$

$B = \{2, 3, 5, 7\}$ $P(B) = \frac{4}{9}$

$C = A \cup B = \{1, 2, 3, 5, 7, 9\}$ $P(A \cup B) = \frac{6}{9} = \frac{2}{3}$

$D = A \cap B = \{3, 5\}$ $P(A \cap B) = \frac{3}{9} = \frac{1}{3}$

$E = A - B = \{1, 9\}$ $P(A \cup B) = \frac{2}{9}$

$F = (A \cup B)^{\backslash} = \{4, 6, 8\}$ $P((A \cup B)^{\backslash}) = \frac{3}{9} = \frac{1}{3}$

Example.2

A die is tossed twice, and the upper face is observed in each time, Find the probability of each of the following events:

A: The event of the occ. of two numbers whose sum is 8.
B: The event of the occ. of two numbers whose difference is 3.
C: The event of the occ. of two numbers whose sum divisable by 3.
D: The event of the occ. of two numbers whose sum divisable by 5.

E: The event of the occ. of two numbers whose sum divisable by 3 and 5.

F: The event of the occ. of two numbers whose sum divisable by 3 or 5.

Solution:

A = { (2,6) , (3,5) , (4,4) , (6,2) , (5,3) } $P(A) = \frac{5}{36}$

B = { (1,4) , (2,5) , (3,6) , (4,1) , (5,2) , (6,3) } $P(B) = \frac{6}{36}$

C = { (1,2) , (1,5) , (2,1) , (2,4) , (3,3) , (3,6) , (4,2) , (4,5) , (5,1) , (5,4) ,

(6,3) , (6,6) } $P(C) = \frac{12}{36} = \frac{1}{3}$

D = { (1,4) , (2,3) , (3,2) , (4,1) , (4,6) , (5,5) , (6,4) } $P(C) = \frac{7}{36}$

$E = E \cap C = \varnothing$ $P(E) = P(\varnothing) = 0$

$F = C \cup D \rightarrow P(F) = P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$$= \frac{1}{3} + \frac{7}{36} - 0 = \frac{19}{36}$$

Example.3

A coin is tossed three consecutive times, and the upper face is observed in each time, find the probability of each of the following events:

A: The event of the occ. of at least one head

B: The event of the occ. of at most one head

C: The event of the occ. of exactly one head

D: The event of the occ. of two consecutive heads

Solution:

S = { (H,H,H) , (H,H,T) , (H,T,H) , (H,T,T) , (T,H,H) , (T,H,T) , (T,T,H) , (T,T,T) }

A = { (H,H,H) , (H,H,T) , (H,T,H) , (H,T,T) , (T,H,H) , (T,H,T) , (T,T,H) }

$P(A) = \frac{7}{8}$

B = { (H,T,T) , (T,H,T) , (T,T,H) , (T,T,T) } $P(B) = \frac{4}{8} = \frac{1}{2}$

C = { (T,H,T) , (T,T,H) , (H,T,T) } $P(B) = \frac{3}{8}$

D = { (H,H,H) , (H,H,T) , (T,H,H) } $P(B) = \frac{3}{8}$

Example.4

A box containing 3 balls one red, one white and one black, two balls are selected at random one after the other with replacement.
calculate the probability of:

i: The two selected balles have the same color.
ii: At least one red ball.
iii: At most one red ball.

Solution:

S = { (b,b) , (b,w) , (b,r) , (r,w) , (r,b) , (r,r) , (w,r) , (w,b) , (w,w) }

i: A = { (b,b) , (r,r) , (w,w) } $P(B) = \frac{3}{9} = \frac{1}{3}$

ii: B = { (b,r) , (r,w) , (r,b) , (r,r) , (w,r) } $P(B) = \frac{5}{9}$

iii: C = { (b,b) , (b,w) , (b,r) , (r,w) , (r,b) (w,r) , (w,b) , (w,w)} $P(B) = \frac{8}{9}$

Example.5

The prob. that Mohamed succeeds in math's exam is 0.75, and the prob. that he succeeds in physics is 0.65, and the prob. that he succeeds in both of them is 0.45, Find

i: The prob. that he succeeds in at least one of them.
ii: The prob. that he fails in both.
iii: The prob. that he succeeds only in maths.
iv: The prob. that he succeeds in at most one of them.

Solution:

let P(maths) = P(A) = 0.75 , P(physics) = P(B) = 0.65
P(both) = $P(A \cap B) = 0.45$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.75 + 0.65 - 0.45 = 0.95$

ii) $P(A \cup B)^{\backslash} = 1 - P(A \cup B) = 1 - 0.95 = 0.05$

iii) $P(A - B) = P(A) - P(A \cap B) = 0.75 - 0.45 = 0.3$

iv) $P(A \cap B)^{\backslash} = 1 - P(A \cap B) = 1 - 0.45 = 0.55$

Example.6

If A, B are two events in S, $P(A) = 0.7$, $P(B) = 0.6$, $P(A \cap B) = 0.4$,

Find: $P(A \cup B)$, $P(A \cap B^{\prime})$, $P(A^{\prime} \cap B^{\prime})$

Solution:

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.4 = 0.9$

$P(A \cap B^{\prime}) = P(A) - P(A \cap B) = 0.7 - 0.4 = 0.3$

$P(A^{\prime} \cap B^{\prime}) = P(A \cup B)^{\prime} = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

Example.7

If A, B are two events in S, $P(A) = 0.6$, $P(B^{\prime}) = 0.6$, $P(A^{\prime} \cup B^{\prime}) = 0.7$,

Find: $P(A-B)$, $P(A^{\prime} \cap B^{\prime})$, $P(A^{\prime} \cap B)$

Solution:

$P(B) = 1 - P(B^{\prime}) = 1 - 0.6 = 0.4$ and,

$P(A^{\prime} \cup B^{\prime}) = P(A \cap B)^{\prime} = 1 - P(A \cap B) = 0.7$

$\Rightarrow P(A \cap B) = 1 - 0.7 = 0.3$

1) $P(A-B) = P(A) - P(A \cap B) = 0.6 - 0.3 = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.4 - 0.3 = 0.7$

2) $P(A^{\prime} \cap B^{\prime}) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

3) $P(A^{\prime} \cap B^{\prime}) = P(A \cup B)^{\prime} = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

Example.8

If A, B are two events in S, $P(A) = 0.7$, $P(A-B) = 0.3$, $P(B^{\prime}) = 0.4$,

Find: $P(A \cup B)$, $P(A^{\prime} \cap B^{\prime})$, $P(A^{\prime} \cap B)$

Solution:

$P(B) = 1 - P(B^{\prime}) = 1 - 0.4 = 0.6$

$P(A-B) = P(A) - P(A \cap B)$

$0.3 = 0.7 - P(A \cap B) \quad \Rightarrow P(A \cap B) = 0.7 - 0.3 = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.4 = 0.9$

$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

$P(A' \cap B) = P(B) - P(A \cap B) = 0.6 - 0.4 = 0.2$

Example.9

A class contains 50 students, 25 of them study chemistry, 20 of them study history and 15 study both. A student is chosen at random. find the pro. that
i) The student studies chemistry or history
ii) The student does not study either of these subjects
iii) The student studies only history

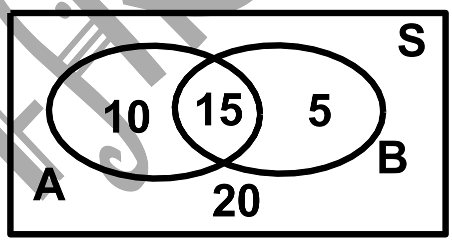
Solution:

chemistry = A $\Rightarrow P(A) = \frac{25}{50} = \frac{1}{2}$

history = B $\Rightarrow P(B) = \frac{20}{50} = \frac{2}{5}$

Both = $A \cap B \Rightarrow P(A \cap B) = \frac{15}{50} = \frac{3}{10}$

another solution



i) The student studies chemistry or history

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{2}{5} - \frac{3}{10} = \frac{3}{5}$

ii) The student does not study either of these subjects

$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{3}{5} = \frac{2}{5}$

iii) The student studies only history

$P(B-A) = P(B) - P(A \cap B) = \frac{2}{5} - \frac{3}{10} = \frac{1}{10}$

Example.10 **Two students try to solve a problem if the prob. that 1st student only solve it is 0.3 and the prob. that the 2nd solve it is o.6 and the prob. that the both of them solve it is 0.4. calculate the prob. that the problem is solvrd by only one of them**

Solution:

$P(A \cap B) = 0.4$, $P(B) = 0.6$

$P(A-B) = P(A) - P(A \cap B)$ $\therefore \Rightarrow$ $0.3 = P(A) - 0.4$ $\therefore \Rightarrow$ $P(A) = 0.7$

$P(A-B) \cup P(B-A) = P(A) + P(B) - 2P(A \cap B)$

$= 0.7 + 0.6 - 2 \times 0.4 = 0.5$

Example.11

Three players A, B, C associated in a swimming race, if the prob. that A wins equals double the prob. that B wins, and the prob. that B wins equals the prob. that C wins. Find the prob. that B or C wins

Solution:

$P(B) = P(C) = x \quad , P(A) = 2x$

$x + x + 2x = 1 \quad \therefore\Rightarrow \quad 4x = 1 \quad \therefore\Rightarrow \quad x = \frac{1}{4}$

$P(A) = 2\times\frac{1}{4} = \frac{1}{2}$ and $P(B) = P(C) = x = \frac{1}{4}$

$P(B\cup C) = P(B) + P(C) - P(B\cap C) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}$

Example.12 **A die is designed such that the prob. of occurrence of the numbers 1,2,3,4,5 are equal, and the prob. of occ. of the number 6 is double the prob. of occ. of the number one, if this die is tossed one time. Calculate the prob. of occ. of an even number.**

Solution:

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$x + x + x + x + x + 2x = 1 \quad \therefore\Rightarrow \quad 7x = 1 \quad \Rightarrow \quad x = \frac{1}{7}$

$A = \text{Even numbers} = \{ 2 , 4 , 6 \} \qquad P(A) = \frac{1}{7}$

Example.13 **If A,B are two mutually exclusive events, P(A - B) = o.7 $P(A\cup B) = 0.9$ Find : $P(B')$, $P(A'\cup B')$, $P(A'\cap B')$**

Solution:

$P(A\cap B) = 0$

$P(A - B) = P(A) - P(A\cap B) \quad \therefore\Rightarrow \quad 0.7 = P(A) - 0 \quad \therefore\Rightarrow \quad P(A) = 0.7$

1] $P(B') = 0.9 - 0.7 = 0.2 \quad \therefore\Rightarrow \quad P(B') = 1 - 0.2 \quad \therefore\Rightarrow \quad P(B') = 0.7$

2] $P(A'\cup B') = 1 - P(A\cap B) = 1 - 0 = 1$

3] $P(A'\cap B') = 1 - P(A\cup B) = 1 - 0.9 = 0.1$

Example.14

If A, B are two events in S, $P(A) = 0.5$, $P(B) = 0.4$, Find $P(A' \cap B')$ in each of the following cases:

i. $P(A' \cup B') = 0.8$

ii. A , B are mutually exclusive events

iii. $B \subset A$

Solution:

i. $P(A' \cup B') = 1 - P(A \cap B)$

$0.8 = 1 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2 = 0.7$

$P(A' \cap B') = 1 - (A \cup B) = 1 - 0.7 = 0.3$

ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 = 0.9$

$P(A' \cap B') = 1 - (A \cup B) = 1 - 0.9 = 0.1$

iii. $B \subset A \quad \therefore \Rightarrow \quad P(A \cup B) = P(A) = 0.5$

$P(A' \cap B') = 1 - (A \cup B) = 1 - 0.5 = 0.5$

Example.15 **If A,B are two events in S, $A \subset B$, given that $P(A) = 0.5$ and the prob. that only B occurrence = 0.2**

Find the prob. that B does not occurred

Solution:

$\therefore A \subset B \quad \therefore \Rightarrow P(A \cap B) = P(A)$ and $P(A \cup B) = P(B)$

$\because$ the prob. that only B occurrence = 0.2 $\therefore \Rightarrow P(B-A) = 0.2$

$P(B-A) = P(A) - P(B)$

$0.2 = P(B) - 0.5 \quad \therefore \Rightarrow P(B) = o.2 + 0.5 = 0.7$

the prob. that B does not occurred = $P(B')$

$P(B') = 1 - P(B) = 1 - 0.7 = 0.3$

Example.16 If A,B are two events in S, $\frac{P(A)}{P(A')} = \frac{3}{2}$, and P(B) = 0.7,

P(A-B) = 0.3 Find $P(A' \cap B')$, $P(A-B')$

Solution:

$2P(A) = 3\,P(A')$ $\quad 2P(A) = 3[1-P(A)] = 3 - 3\,P(A)$

$5P(A) = 3$ $\quad P(A) = 3/5 = 0.6$ and $\quad P(B) = 0.7$

$P(A-B) = P(A) - P(A \cap B)$

$0.3 = 0.6 - P(A \cap B) \quad \therefore \Rightarrow P(A \cap B) = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = O.6 + 0.7 - 0.4 = 0.9$

$P(A' \cap B') = 1 = P(A \cup B) = 1 - 0.9 = 0.1$

$P(A-B') = P(A \cap B) = 0.3$

Example.17 If A,B are two events, in S $P(A) = 2P(A')$, and P(B) = 0.5 ,

$P(A \cap B) = 0.25$, find $P(A \cup B)$, $P(A \cap B')$, $P(A-B')$, $P(A'-B')$

Solution:

$P(A) = 1 - P(A') \quad \therefore \Rightarrow \quad 2P(A') = 1 - P(A')$

$3P(A') = 1 \quad \therefore \Rightarrow \quad P(A') = 1/3$ and $P(A) = 2/3$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = (2/3) + (1/2) - (1/4) = 11/12$

ii) $P(A \cap B') = P(A) - P(A \cap B) = (2/3) - (1/4) = 5/12$

iii) $P(A-B') = P(A') - P(A \cap B') = (2/3) - (5/12) = 1/4$

iv) $P(A'-B') = P(B-A) = P(B) - P(A \cap B) = 0.5 - 0.25 = 0.25$

Home work

1. A student wants to buy a bag. It is possible to choose from three types. Each one has two sizes and the color of the bag is either black or brown. Represent the sample space by a tree diagram.

2. In an experiment of tossing a coin once, then a die is rolled, observing the upper faces.

   a Write down the sample space of this experiment, then determine the following events.

   - A: appearance of a head and an odd number.
   - B: appearance of a tail and an even number
   - C: appearance of a prime number > 2
   - D: appearance of a number divisible by 3

3. A die is rolled two consecutive times, the number on the upper face is observed in each time Determine each of the following events:

   - A: The appearance of two equal numbers
   - B: The appearance of two numbers their sum equals 9
   - C: The appearance of two numbers their sum equals 13
   - D: The appearance of number 3 one time at least.

4. From the set of numbers {1, 2, 3, 4} we need to form a two different digit number. Represent the sample space in a tree diagram, and then determine the following events:

   - A: The event "the unit digit is an odd number ".
   - B: The event "the tens digit is an odd number".
   - C The event "the two digits are odd numbers".
   - D The event "the unit digit or the tens digit is an odd number.

5. A bag contains 20 identical cards numbered from 1 to 20, If a card is selected randomly and the number written on it is recorded. Write the following events:

   A the event "the recorded number is even and greater than 10

   B the event "the recorded number is a factor of 12"

   C the event "the recorded number is odd and divisible by 3

   D the event "the recorded number is a multiple of the two numbers 2, 5

   E the event "the recorded number is prime"

   F the event "the recorded number satisfying the inequality $5x - 3 \leqslant 17$

6. Two cards are drawn one after the other from a set of 8 identical cards numbered from 1 to 8 and the drawn card must returned before drawn another card. What is the number of the elements in the sample space? and if:

   A: is the event "the number in the second draw is three times the number in the first draw"

   B: is the event "the sum of the two numbers is more than 13"

   Write the events A, B. Are there two mutually exclusive events? Explain that.

7. In the experiment of tossing a coin three consecutive times and observing the sequence of heads and tails .represent the sample space with tree diagram, then determine the following events:

   A the event "appearance of two tails at least" B the event "appearance of two tails at most"

   C the event " appearance of a head in the first toss"

   A the event " non-appearance of a head in the three tosses"

8. In an experiment of tossing a coin once, then a die is rolled, observing the upper faces Represent the sample space of this experiment by a probability tree diagram, and then determine the following events:

   A appearance of a Tail and an even number"

   B appearance of a head and an odd number"

   C non-occurrence of A or non- occurrence of B"

   D occurrence of the event A only

   E occurrence of the event A and occurrence of the event B

Choose the correct answer from those given :

9. If a regular die is rolled once, then the probability of the appearance of an odd number less than 5 in the upper face equals:

   a $\frac{2}{5}$ b $\frac{1}{2}$ c $\frac{1}{3}$ d $\frac{1}{6}$

10. If a regular die is rolled twice, then the probability of the appearance of an even number in the first roll and a prime number in the second roll equals:

    a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{1}{9}$ d $\frac{1}{4}$

11. If a ball is drawn randomly from a box contained 3 white balls, 5 red balls and 7 green balls, then the probability that the selected ball is white or green equals:

    a $\frac{1}{5}$ b $\frac{2}{3}$ c $\frac{7}{15}$ d $\frac{1}{2}$

12) A card is drawn from a set of 9 identical cards numbered from 1 to 9. What is the probability that the drawn card carrying a divisor of (factor of) 9 or an odd number equals:

a $\frac{1}{3}$ b $\frac{7}{9}$ c $\frac{1}{2}$ d $\frac{5}{9}$

13) If A , B are two events in a sample space of a random experiment $B \subset A$, $P(A) = 2P(B) = 0.6$ then $P(A - B)$ equals:

a 0.6 b 0.3 c 0.4 d 0.2

14) A uniform die, the numbers 8, 9, 10, 11, 12, 13 written in its faces. If the die is rolled once, observing the number appearing on its upper face

a Find the probability of each of the following events:

- A: "appearance of an odd number."
- B "appearance of a prime number."
- C: "appearance of an even number."
- D "appearance of a number great than 12."
- E: "appearance of a number consists of two digits."
- F "appearance of a number consists of only one digit."

b Calculate: $P(A \cup C)$, $P(E \cup F)$, $P(B \cap D)$.

15) If is a sample space of a random experiment, where S = {A , B, C, D}, find: P(A) , P(B) , given that $P(A) = 3P(B)$, $P(C) = P(D) = \frac{7}{18}$

16) If A, B are two mutually exclusive events, S is a sample space of its random experiment, If $P(A \cup B) = 0.6$, $P(A - B) = 0.25$ find, P(A), P(B).

17) If A, B is a sample space of a random experiment, and $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{4}$ find:

a $P(A')$ b $P(A \cup B)$ c $P(A - B)$ d $P(A' \cap B')$

18) If A, B are two events, of a sample space of a random experiment, where: $P(A) = 0.4$, $P(B') = 3P(B)$, $P(A \cap B) = 0.2$ find the probability of:

a Occurrence of A only. b Occurrence of A or B

c Occurrence of A and non-Occurrence of B.

19) A box contains colored identical balls: 4 red, 6 blue, and 5 yellow. A ball is selected at random from the box, find the probability that the drawn ball is:

a Red. b Blue or yellow. c Not blue. d Not red and not yellow.

20) One card is selected at random from 30 identical cards numbered from 1 to 30. Find the probability that the selected card is carring a number:

a Divisible by 3 b divisible by 5

c Divisible by 3 and 5 d divisible by 3 or 5

21. Three distinct coins are tossed once. Observing the upper faces, find the probability of the following events:

- A: appearance of a head or two heads.
- B: appearance of at least one head.
- C: appearance of a head at most.
- D: appearance of at two consecutive tails at least.

22. A die is rolled two consecutive times, the number on the upper face is observed in each time. Find the probability of each of the following events:

- Appearance of the number 4 in the first roll.
- Appearance of two numbers, their sum equals 8
- Appearance of two numbers, their sum is less than or equal 5

23. **Join with sport :** A random sample consists of 60 persons in a survey, it is found that 40 of them encourage Al Hilal club, 28 of encourage El negma club and 8 of them don't encourage any of them. A person is chosen at random from the sample. Find the probability that the chosen person encourages:

a. At least one of the two clubs.
b. Both clubs.
c. Al Hilal club only.
d. Only one of the two clubs.

24. In an experiment of tossing a coin once, then a die is rolled once , observing the upper faces. If A is the event of the appearance of a head and a prime number, B is the event of the appearance of an even number. Find the probability of the occurrence of each of the two events, and then calculate the probability of the following events:

a. The occurrence of one of the events at least
b. The occurrence of the two events together
c. The occurrence of only the event B
d. The occurrence of only one of the two events

25. a card is selected randomly from 50 identical cards numbered from 1 to 50, if the number written on it is recorded. Find the probability that the number written on the selected card is:

a. A multiple of number 7
b. A perfect square number
c. A multiple of number 7 and a perfect square number
d. Not a perfect square number and not a multiple of 7

26. If A , B are two events, in a sample space of a random experiment, where: $P(B) = \frac{4}{5} P(A)$, $P(A - B) = 0.24$ $\quad P(B \cap A') = 0.15$ then find: $P(A)$, $P(B)$, $P(A \cup B)$, $P(A' \cup B')$

27. Tarek wrote 75 letters on the typewriter ,he found that 60% of them are without mistakes and Zead wrote 25 letters on the typewriter ,he found that 80% of them are without mistakes. If a letter is selected randomly from all letters written by both Tarek and Zead ,then find the probability that the selected letter is:

a. Without mistakes.
b. Written by Zead.
c. Written by Zead without mistakes.
d. Written by Tarek with mistakes.

28. If A , B are two events, in a sample space of a random experiment, where: $P(A) = 0.6$, $P(B) = 0.8$, $P(A' \cup B') = 0.5$ then find $P(A' \cap B)$

General Exercise on (unit four)

Choose the correct answer from those given:

In an experiment of rolling a die twice then:

(1) The probability of the appearance of number 5 in the first roll and number 6 in the second roll is:

(a) $\frac{1}{24}$ (b) $\frac{1}{30}$ (c) $\frac{1}{36}$ (d) $\frac{1}{6}$

(2) The probability of the appearance of number 5 in one of the two rolls and the number 6 in the other roll is:

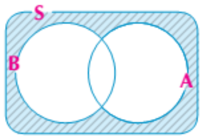
(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{5}{36}$ (d) $\frac{1}{18}$

(3) The probability of the appearance of two equal numbers in the two rolls

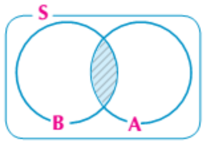
(a) $\frac{1}{5}$ (b) $\frac{1}{36}$ (c) $\frac{1}{6}$ (d) $\frac{1}{18}$

If A and B are two events in a sample space of a random experiment, choose the event that represents the shaded parts in the opposite venn diagram :

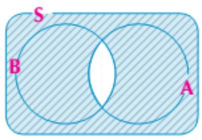
(4) (a) $S - (A \cup B)$ (b) $A' \cup B'$ (c) $S - (A' \cup B')$ (d) $(A \cap B)'$



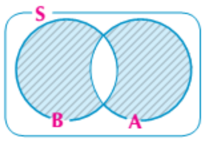
(5) (a) $A \cap B$ (b) $A \cup B'$ (c) $(A \cup B)'$ (d) $A' \cup B'$



(6) (a) $(A \cup B) - (A \cap B)$ (b) $S - (A \cap B)'$ (c) $S - (A \cup B)'$ (d) $S - (A \cap B)$



(7) (a) $S - (A \cap B)$ (b) $A' \cap B'$ (c) $(A - B) \cup (B - A)$ (d) $S - (A \cup B)'$



(8) Form two digit number from the digits of the number 4321 , and represents the sample space using the probability tree diagram ,then write the sample space and the following events:

A: event" the set of prime numbers"

B: event" the set of numbers that divisible by 3"

C: event" the set of numbers that divisible by 3 and 5"

D: event" the set of numbers that its unit digit is twice of its tens digit"

(9) In the experiment of rolling a die once and observing its upper face .Find the probability that the appearance number is:

(a) A prime number (b) A factor of 6 (c) An odd number and divisible by 3

10 If A, B are two events in a sample space S of a random experiment.

Complete each of the following to get true statement:

a If A, B are two mutually exclusive events:

- $A \cap B =$ ....
- $A - B =$ ....
- $B - A =$ ....

b If $B \subset A$ then:

- $(A \cap B) =$ ....
- $A \cup B =$ ....
- $B - A =$ ....

c If $P(A) = 0.3$, $P(B) = 0.5$, $P(A \cap B) = 0.1$ then:

- $P(A \cup B) =$ ....
- $P(B - A) =$ ....
- $P(A' \cap B') =$ ....

d If $A \cap B = \phi$, $P(A') = 0.7$, $P(B') = 0.4$ then:

- $P(A \cup B) =$ ....
- $P(A \cap B) =$ ....
- $P(A - B) =$ ....

11 If A, B are two events in a sample space S of a random experiment, $A \subset B$, $P(A) = \frac{1}{2}$, and probability of occurrence of B only = 0.2. Calculate the probability of non-occurrence of B.

12 If A, B are two events in a sample space S of a random experiment , $P(A) = \frac{1}{2}$, , $P(B) = x$ and $P(A \cup B)' = \frac{1}{3}$

a Find the value of x in each of the following:

- If A, B are two mutually exclusive events.
- $A \subset B$.

b If $x = \frac{1}{4}$ Find the value of $P(A \cap B)$.

13 If A, B are two events, of a sample space S of a random experiment, write a symbolical expression of each of the following events, representing them by Venn -diagrams:

a Non occurrence of A

b Occurrence of A or B.

c Occurrence of B only.

d Occurrence of A or non-occurrence of B.

e Non occurrence of both of them together.

f Occurrence of only one of them.

14 A coin is tossed three consecutive times, observing the upper faces, represent the sample space by a tree diagram. Find the probability of the following events:

a A: appearance of only two heads

b B: appearance of at most two heads

c C: appearance of at most one tail

d D: appearance of same outcomes in the three tosses

15 A die is tossed two consecutive times, the number on the upper face is observed in each time.

a Draw a tree diagram that represents the sample space, explaining the following events on it.

- A: appearance of two numbers, their sum is an odd number which is greater than 6
- B: appearance of two numbers , one of them is 2 and their sum $\leqslant 5$
- C: appearance of two equal numbers

b Are A, B and C two by two mutually exclusive events?

c Calculate the probability of each of the following: $P(A \cup B)$, $P(B \cap C)$, $P(A \cup C)'$, $P(B - C)$.

16 Five identical cards numbered from 2 to 6, two cards are drawn randomly one after the other with replacement and observing the sequence of the numbers on the selected cards to form all possible two digit numbers. Find the probability that:

a its unit digit is a prime number.

b its tens digit is an odd number.

c its unit digit is a prime number or its tens digit is an odd number.

Newspaper:

17 In a sample of 50 persons, it is found that 27 of them read newspaper (A), 24 of them read newspaper (B) and 9 persons read both newspapers. A person is chosen at random from this sample. Find the probability that the chosen person read:

a Newspaper A only.

b At least one of the two newspapers.

Tourism:

18 In one of the sound and light shows at the pyramids, 200 people attend from various nationalities, the opposite table shows their data. If one of them was chosen at random by their access cards to give him the souvenir prize, then: Find the probability that the chosen person is :

	Arab	European	American	Total
Man	32	47	15	94
Woman	23	63	20	106
Total	55	110	35	200

a An European man.

b An American Woman.

c A Woman.

d With Arab or European nationality.

19 If S = {A, B, C}, is a sample space of a random experiment and, $20P(A) = 15P(B) = 12P(C)$

Find: $P(A)$, $P(B)$, $P(C)$